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**MEASUREMENT ERRORS PROCESSING
BY COVARIANCE ANALYSIS FOR AN
IMPROVED ESTIMATION OF DRYING
CHARACTERISTIC CURVE PARAMETERS**

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ABSTRACT

The aim of this paper is to show the interest of the covariance analysis applied to measurement error in the particular case of the identification of a drying characteristic curve from experimental drying data. The modelisation of drying by use of the Drying Characteristic Curve (DCC) method is first presented with usual specifications (power function, critical moisture content...). The experimental procedure used to obtain drying curves and the data processing are detailed and analysed. Measurements errors are identified at the first step of the

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43 procedure and their effects on the estimation error of the expo-
44 nent α of the power function are estimated. Three different
45 methods for estimating α are presented under their matrix
46 form: the least square method and two methods based on the
47 «Gauss–Markov» or «Maximum likelihood» theorem, firstly
48 under a simplified form suited if the estimation errors are
49 uncorrelated and secondly under a complete form suited even
50 if the estimation errors are correlated. These three
51 methods are applied to experimental results obtained with
52 ginger roots drying. The value of the exponent α of the
53 power function and then the distances between the three
54 corresponding theoretical drying curves (representing product
55 water content vs. time) and the experimental points are studied.
56 It is shown that in this particular application, the complete
57 Gauss–Markov method leads to the better fitting and that
58 the simplified Gauss–Markov method, since it is a priori non
59 applicable in this case where errors are correlated, gives quite
60 better results than the ordinary least squares method. The cov-
61 ariance matrices of the estimation errors of reduced water con-
62 tent, reduced drying rate and exponent α are also presented in
63 order to show the correlations existing between the measure-
64 ment errors of each variable during a drying cycle.

65
66 *Key Words:* Drying characteristic curve; Inverse method;
67 Covariance analysis; Measurement error processing
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69

70 INTRODUCTION

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72 The «Drying Characteristic Curve» (DCC) is a concept proposed by
73 Van Meel^[1] in 1957. It consists in processing and reducing drying experi-
74 mental results. The aim of such a processing is to obtain a single drying
75 curve for a given product with given sizes, independently of the drying air
76 conditions (temperature, humidity, speed), by use of convenient variables
77 reductions applied to the product water content and to the drying rate.

78 Following Van Meel, many authors among them Desmorieux and
79 Moyne,^[2] Belahmidi et al.^[3] Fornell et al.^[4] Salgado et al.^[5] and
80 Ahouannou et al.,^[6] have used this concept to characterize product proper-
81 ties with regard to drying.

82 Nevertheless, such authors do not describe deeply the way they
83 take into account the measurement errors related to parameters estimation.
84 Therefore, the aim of this paper is to show the interest of processing

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85 measurement errors by covariance analysis that can include the following
86 aspects:

- 87
- 88 – Improved estimation of the DCC parameters.
 - 89 – The study of the error propagation and sensitivity to DCC
90 parameters can give information on the way to obtain the most
91 confident result.

92 In this paper, the procedure to obtain the Drying Characteristic Curve
93 (DCC) is first presented and analysed. The experimental procedure used to
94 obtain drying curves and the mathematical calculation leading to the DCC
95 from them are detailed. Error measurements at each step of the procedure
96 and their effects on the estimation error of the exponent α of the power
97 function are then described. Three different methods for estimating α are
98 presented under their matrix form: the least square method and two
99 methods based on the Gauss–Markov theorem. These three methods are
100 applied to experimental results related to ginger roots drying in order to
101 determine the value of the exponent α . The error propagation is analysed by
102 the study of the covariance matrices of the estimation errors of reduced
103 water content, reduced drying rate and exponent α . A sensitivity analysis
104 of the different parameters is finally proposed.

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107 **THE DRYING CHARACTERISTIC CURVE METHOD**

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109 The «Drying Characteristic Curve» represents the reduced drying rate
110 V_r as a function of the reduced water content X_r . These two variables are
111 defined as follows:

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$$V_r(t) = \frac{V(t)}{V_1} \quad \text{and} \quad X_r(t) = \frac{X(t) - X_{eq}}{X_{cr} - X_{eq}} \quad \text{with} \quad V = \frac{dX}{dt}$$

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A mathematical expression of the DCC for a product with fixed initial sizes is sought by analyzing the experimental drying curves obtained with various drying air temperature, humidity and speed conditions. One representative example of a presentation such as $dX/dt = f(X)$ is shown in Fig. 1.

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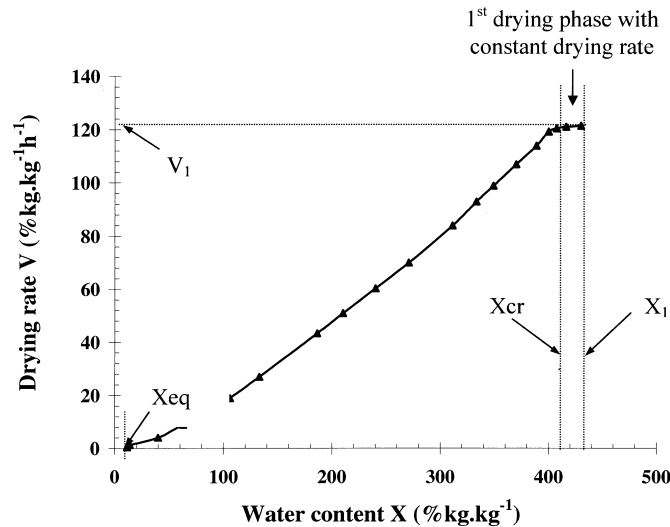


Figure 1. Example of drying curve $V=f(X)$ for mango with drying air at $T=40^{\circ}\text{C}$, $HR=15\%$ and $v=1\text{ m s}^{-1}$.

For biological products, the raising temperature phase is most often negligible, especially if the difference between air and product temperatures is low and if the product dimensions are small. Our experimental results have confirmed this hypothesis.

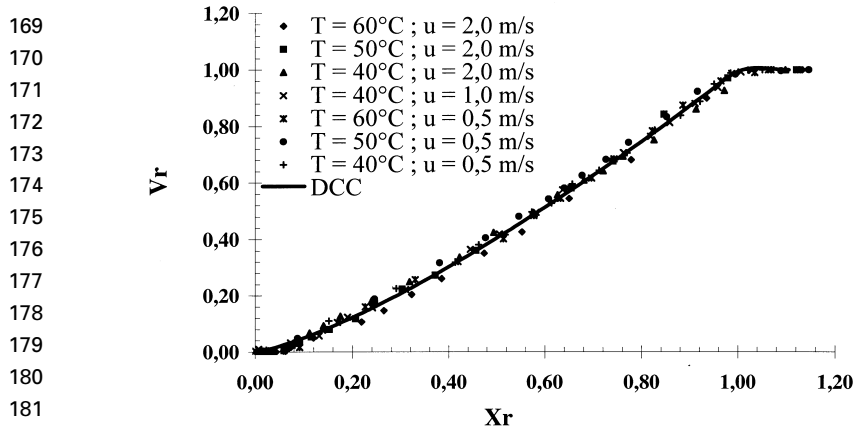
The experimental curves $X(t)$ have been derived to obtain the estimated curves $V(t)=dX/dt(t)$. Such analysis gives a mean value of the critical water content X_{cr} as shown in Fig. 1. Several authors (among them Desmorieux and Moyne^[2]) have considered that for biological products it is difficult to identify a critical water content different from the initial water content X_1 . Nevertheless, Talla et al.^[7] could identify a critical water content $X_{cr} \neq X_1$ for banana and mango.

Then, from each experimental point (t, X) , the couples of corresponding values (X_r, V_r) are calculated and the whole points obtained for various drying air conditions are placed on a unique graph representing $V_r=f(X_r)$ to obtain a cloud of points illustrated in Fig. 2.

A mathematical expression of the DCC: $V_r=f(X_r)$ which fit this cloud of points more or less dispersed is then sought. The function f having to respect the following conditions:

- If $X=X_{eq}$: $X_r=0$ and $V=0$ so $V_r=0$ and finally $f(0)=0$
- If $X=X_{cr}$: $X_r=1$ and $V=V_1$ so $V_r=1$ and finally $f(1)=1$
- $0 < f(X) < 1$ if $0 < X_r < 1$.

F2



185 *Figure 2.* Drying characteristic curve for mango.

186 The function f is more often sought as a power function (Ahouannou
187 et al.^[6] and Talla et al.^[7]) It also could be seek as a polynomial expression as
188 noted by Desmorieux and Moyne.^[2] In this study, the case of a power
189 function $V_r = X_r^\alpha$ has been considered. This function is written in a devel-
190 oped form as:

$$191 \frac{V(t)}{V_1} = \left[\frac{X(t) - X_{eq}}{X_{cr} - X_{eq}} \right]^\alpha \quad (1)$$

192
193 Experiments have been conducted within the drying tunnel depicted in
194 Fig. 3, the objectives were:

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- 196 – Obtaining experimental drying curves $X=f(t)$ for given products
197 and air conditions
- 198 – Determining of the value of the exponent α which ensure the best
199 fitting between the experimental points $X_{(t)}$ and the theoretical
200 curves obtained by integration of Eq. (1):

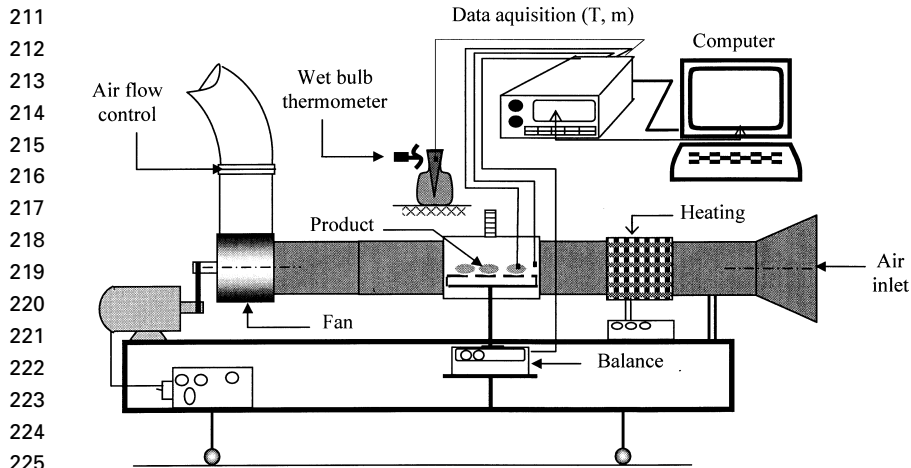
201
202 If $t \leq t_c$ where $t_c = \frac{X_{cr} - X_1}{V_1}$: $X(t) = X_1 + V_1 t$

203
204 If $t > t_c$ if $\alpha \neq 1$:

$$205 X(t) = X_{eq} + (X_{cr} - X_{eq}) \left[1 + \frac{(1 - \alpha)V_1(t - t_c)}{(X_{cr} - X_{eq})} \right]^{1/(1-\alpha)}$$

206
207 if $\alpha = 1$: $X(t) = X_{eq} + (X_{cr} - X_{eq}) \exp \left[\frac{V_1(t - t_c)}{X_{cr} - X_{eq}} \right]$

208
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210 where: t_c First drying time duration.



226 *Figure 3.* Schematic view of the drying tunnel.

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229 **EXPERIMENTAL DETERMINATION OF**
230 **THE VARIOUS VARIABLES**

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232 It is considered here that the result \hat{X} of a measurement or estimation
233 of a parameter X is the sum of the true or exact value of X plus a random
234 variable eX : $\hat{X} = X + eX$. The random variable eX is called the measure-
235 ment error, the average of the k values eX_i ($i = 1, \dots, k$) obtained by repeat-
236 ing k times the measurement of X is considered to be equal to zero. What is
237 usually called measurement uncertainty is a value dX such as the absolute
238 values of eX_i are always lower than dX . So that dX can also be seen as the
239 maximum observable value of the error eX . If the value \hat{X} of the parameter
240 X is directly obtained by use of a measuring apparatus, dX is called the
241 precision of the apparatus.

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245 **Product Initial Water Content X_1 Determination**

246 The fresh product (before any drying) water content is calculated as:

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$$X_1 = \frac{m' - m'_d}{m'_d}$$

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251 where m' is the mass of a piece of fresh product representative of the product
252 to be dried and m'_d its bone dry mass. Both masses are estimated by

253 difference between the mass of the product + support set and the mass of the
 254 support, this last one being measured before disposing the piece of fresh
 255 product on it. So, the measurement uncertainties dm' and dm'_d on the masses
 256 m' and m'_d obtained by the difference of two weighed masses is twice the
 257 precision Δm of the balance.

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Product Water Content $X(t)$ Determination

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A mass m_1 of fresh product is then placed on a support which mass M
 is determined by weighting. The product + support set mass $(M + m)_1$ is
 measured before introduction in the drying tunnel. The temperature, humid-
 ity and speed of the air flow are regulated in order to obtain constant
 conditions. The mass of the support + product set is measured at various
 times t_i until the product water content reached the desired final value. The
 product water content at time t_i is calculated as:

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$$X_i = \frac{m_i - m_d}{m_d}$$

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where:

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$$m_i = (M + m)_i - M$$

$$m_d = \frac{m_1}{(1 + X_1)}$$

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Drying Rate $V(t)$ Determination

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The drying rate V_i is deduced from the experimental points (t_i, X_i)
 using classical finite difference expressions:

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– First Point ($t = 0$):

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$$V_1 = \frac{X_1 - X_2}{t_2}$$

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– Last point ($t = t_p$):

$$V_p = \frac{X_{p-1} - X_p}{t_p - t_{p-1}}$$

- 295 – Intermediary point ($t = t_i$):

$$296 \quad V_i = \frac{1}{2} \left(\frac{X_{i-1} - X_i}{t_i - t_{i-1}} + \frac{X_i - X_{i+1}}{t_{i+1} - t_i} \right)$$

300 Exponent α Determination

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302 For each measurement at each time t_i , α_i is evaluated by use of the
303 relation:

$$304 \quad \alpha_i = \frac{\ln(V_{r_i})}{\ln(X_{r_i})} \quad \text{with } V_r = \frac{V_i}{V_1} \quad \text{and} \quad X_r = \frac{X_i - X_{\text{eq}}}{X_{\text{cr}} - X_{\text{eq}}}$$

308 MEASUREMENT OR ESTIMATION ERRORS

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310 The following hypotheses have been considered:

- 311
312 – The measurement errors of times t_i at which measurements are
313 realised are negligible.
314 – The estimation error on the evaluated values of X_{eq} and X_{cr} are
315 negligible. This hypothesis concerning X_{cr} will be further justified.
316 – All the mass measurements are done with the same balance which
317 precision is Δm .

318
319 The error calculation for the various variables leads to the following formula:

$$320 \quad eX_1 = \frac{1}{m'_d} [em' + (X_1 - 1)em'_d]$$

$$321 \quad eX_i = (X_i + 1) \left(\frac{em_i}{m_i} + \frac{em_1}{m_1} + \frac{eX_1}{1 + X_1} \right) \quad \text{for } i \neq 1$$

$$322 \quad eX_{r_i} = X_{r_i} \left(\frac{eX_i}{X_i - X_{\text{eq}}} + \frac{eX_1}{X_1 - X_{\text{eq}}} \right)$$

$$323 \quad eV_1 = \frac{eX_1 - eX_2}{t_2 - t_1}$$

$$324 \quad eV_p = \frac{eX_{p-1} - eX_p}{t_p - t_{p-1}}$$

$$325 \quad eV_i = \frac{1}{2} \left(\frac{eX_{i-1} - eX_i}{t_i - t_{i-1}} + \frac{eX_i - eX_{i+1}}{t_{i+1} - t_i} \right)$$

$$326 \quad eV_{r_i} = V_{r_i} \left(\frac{eV_1}{V_1} + \frac{eV_i}{V_i} \right)$$

$$327 \quad e\alpha_i = \frac{eV_{r_i}}{V_{r_i} \ln(V_{r_i})} + \frac{\ln(V_{r_i}) eX_{r_i}}{[\ln(X_{r_i})]^2}$$

337 The measurement uncertainties can be calculated by this formulas by
 338 taking as error measurement on the masses their uncertainties measurement
 339 that is $2\Delta m$, where Δm is the precision of the balance.

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EXPERIMENTAL DATA ANALYSIS METHOD

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A polynomial relationship is the most usual form sought in physical problems, in this case:

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$$\alpha = b_n t^n + b_{n-1} t^{n-1} + \dots + b_1 t + b_0$$

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The relation between the exact value α_i of α at time t_i can be written under the form:

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$$\begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} 1 & t_2 & \dots & t_2^n \\ 1 & t_3 & \dots & t_3^n \\ \vdots & \vdots & & \vdots \\ 1 & t_p & \dots & t_p^n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix}$$

363

Or under a matrix form:

364

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$$[\alpha] = [S][B] \text{ where } [S] = \left[\left(\frac{\partial f}{\partial b_i} \right)_{t_j} \right] \text{ is the sensitivity matrix.}$$

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The determination of the matrix $[B]$ when the matrix $[S]$ is known without error and the matrix $[\hat{\alpha}]$ is known with unnegligible errors can be done using one of the two followings methods:

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The Ordinary Least Square Method

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This classical method as described by Trigeassou^[8] and Press et al.^[9] among many authors can be applied to calculate the vector $[B]$ which minimise the quadratic error D between the measures values vector $[\hat{\alpha}]$ and the

379 exact values vector $[\alpha]$, D being calculated as follows:

$$380 \quad D = \|[\hat{\alpha}] - [S][B]\| = ([\hat{\alpha}] - [S][B])'([\hat{\alpha}] - [S][B])$$

382 The vector B which minimise D is given by:

$$383 \quad [B] = ([S]'[S])^{-1}S'[\hat{\alpha}]$$

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388 Gauss–Markov Method

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390 According to Beck and Arnold,^[10] if the following conditions are
391 verified:

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- 393 – The matrix $[S]$ is known without error,
- 394 – The errors on α_i have zero mean,
- 395 – The covariance matrix of measurement error is known with excep-
396 tion of a multiplicative constant a , so that a matrix P is known that
397 verifies $[P] = a [\text{cov}(e\alpha)]$,
- 398 – The matrix $[P]$ is positive definite,

399 then the vector minimising the difference:

$$400 \quad D = ([\hat{\alpha}] - [S][B])'[\text{cov}(e\alpha)]^{-1}([\hat{\alpha}] - [S][B])$$

402 is given by:

$$404 \quad [B] = ([S]'[P]^{-1}[S])^{-1}[S]'[P]^{-1}[\hat{\alpha}]$$

405

406 This evaluation method called “Gauss–Markov method” take into
407 account the differences between the measurement errors of the various α_i
408 values and minimise the importance of the α_i values with a great measure-
409 ment error. The least square method considers implicitly that all the α_i
410 values have the same measurement errors.

411 The most important problem linked to the application of this method
412 is the evaluation of the matrix $[\text{cov}(e\alpha)]$. Two cases may occur:

413 Simplified method: the measures of α are not correlated i.e., the
414 measurement error of α_i is independent of the measurement error of α_j . In
415 this specific case: $\text{cov}(e\alpha_i, e\alpha_j) = 0$ if $i \neq j$ and the matrix $[\text{cov}(e\alpha)]$ is diagonal:

416

$$417 \quad [\text{cov}(e\alpha)] = \begin{bmatrix} \text{var}(e\alpha_1) & & & \\ & \text{var}(e\alpha_2) & & 0 \\ & & \ddots & \\ 0 & & & \text{var}(e\alpha_p) \end{bmatrix}$$

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COVARIANCE ANALYSIS OF DCC PARAMETERS

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421 Whether the hypothesis that α_i measurement is errorless, the mean
 422 value $E(e\alpha_{ij})$ at i constant of the measurement errors ($e\alpha_{ij}$) is equal to zero
 423 and $\text{var}(e\alpha_i)$ may be written as:

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$$425 \quad \text{var}(e\alpha_i) = \frac{1}{k} \sum_{j=1}^k [e\alpha_{ij} - E(e\alpha_{ij})]^2 = \frac{1}{k} \sum_{j=1}^k (e\alpha_{ij})^2 = \frac{1}{k} \sum_{j=1}^k (r_j d\alpha_i)^2$$

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where r_j is a random number bounded by -1 and $+1$.

With the hypothesis that all the measured values α_{ij} of α at time t_i are dispersed in the same manner around the mean value α_i , it may be written:

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and the covariance matrix take the following form:

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$$441 \quad [\text{cov}(e\alpha)] = a \begin{bmatrix} (d\alpha_1)^2 & & & \\ & (d\alpha_2)^2 & & 0 \\ & & \ddots & \\ 0 & & & (d\alpha_p)^2 \end{bmatrix}$$

Complete method: The measured values of α are correlated i.e., measurement error of α_i is related to measurement error of α_j , so that the previous method cannot be applied. Nevertheless, the information contained in measurement uncertainty value dm_i and in the errors calculus formula may be used in the following way:

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k measurements of $X_1, m_1, m_2, \dots, m_p$ are simulated by giving the value $\hat{X}_1 = X_1 + r_j dX_1$ to the j th simulated value of X_1 and the value $\hat{m}_{ij} = m_i + r_{ij} dm_i$ to the j th simulated value of m_i , where r_j and r_{ij} are random numbers bounded by -1 and $+1$. Using the previously established errors formula, the values $X_{ij}, V_{ij}, X_{r_{ij}}, V_{r_{ij}}, \alpha_{ij}$ corresponding to the masses \hat{m}_{ij} and the errors $eX_{ij}, eV_{ij}, eX_{r_{ij}}, eV_{r_{ij}}$ and $e\alpha_{ij}$ induced by the error em_{ij} are evaluated for i varying from 1 to p and for j varying from 1 to k . A matrix $[e\alpha]$ of the simulated values $e\alpha_{ij}$ is thus obtained which enables an evaluation of the matrix $[\text{cov}(e\alpha)]$ and the application of the Gauss–Markov method. The above described methods may be applied to relations other than polynomial on condition that this relation be linear toward b_i that is to say $(\partial/\partial b_i)[\partial f(t)/\partial b_j] = 0$, for all values of i varying

463 from 1 to p and of j varying from 1 to k (the sensibility matrix S must not
464 depend on b_i).

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Method Comparison

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470 The previously described methods have been applied for the
471 processing of experimental data of a drying cycle: the results of a drying
472 experiment of ginger roots cut in 50 mm long pieces and disposed in an air
473 flowing at $T=40^\circ\text{C}$, $v=1$ m/s and $\text{HR}=32\%$. A first drying phase with a
474 constant drying rate could not be identified so the critical water content X_{cr}
475 is considered as equal to the initial water content X_1 . The equilibrium water
476 content X_{eq} has been calculated by applying Ahouannou et al.^[6] results that
477 leads to the value $X_{\text{eq}}=0.1$ kg kg⁻¹. A balance with a precision of 0.01 g has
478 been used for weight measurements, the experimental results are presented
479 in Table 1.

T1

480 The initial water content (dry basis) value of $X_1=4.559$ kg/kg has been
481 obtained by complete dehydration of a separate sample (representative
482 of the product to be dried) with a final bone dry mass $m_d=30$ g.
483 The calculation of the X_i , V_i , V_{r_i} , X_{r_i} , α_i values and of their uncertainties
484 leads to the results presented in Table 2, the measurement uncertainties
485 of the various masses being equal to 0.02 g that is twice the balance
486 precision.

T2

487 The problem to be solved is the simplest one where a constant and
488 time independent value α is sought which correspond to the estimation of
489 $\alpha=f(t)=b_0$. In this case, the matrix $[S]$ is a column matrix with p rows and
490 each element being equal to 1:

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Table 1. Experimental Values of the Ginger Mass During Its Drying

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499	t_i (h)	0	0.17	0.25	0.42	0.5	0.75	1	1.25	1.5
500	m_i (g)	46.30	44.11	43.19	42.02	41.09	38.48	36.43	34.30	32.48
501	t_i (h)	1.75	2	2.5	3	4	5	6	7	8
502	m_i (g)	30.71	29.07	26.08	23.67	20.11	17.46	15.49	14.06	13.13
503	t_i (h)	10	12	14	16	18	20	22		
504	m_i (g)	11.7	11.02	10.43	10.12	9.87	9.79	9.64		

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505 **Table 2.** Calculated Values of the Water Contents X and X_r , the Drying Rates V
 506 and V_r , the Exponent α and Their Uncertainties During the Drying

507									
508	X_i	$10^4 V_i$							
509	t_i (s)	(kg kg ⁻¹)	(kg kg ⁻¹ s ⁻¹)	X_{r_i}	V_{r_i}	α_i	$10^3 dX_{r_i}$	$10^{-1} dV_{r_i}$	$d\alpha_i$
510	0	4.559	4.296	1	1	—	0	0	—
511	612	4.296	4.066	0.9408	0.9347	1.107	2.342	1.193	2.111
512	900	4.186	3.065	0.9165	0.7074	3.972	2.301	1.179	1.999
513	1512	4.045	3.086	0.8842	0.7122	2.758	2.249	1.161	1.376
514	1800	3.933	3.679	0.8597	0.8429	1.131	2.207	1.118	0.880
515	2700	3.620	3.108	0.7894	0.7202	1.388	2.091	0.651	0.396
516	3600	3.374	2.788	0.7342	0.6460	1.414	2.000	0.608	0.315
517	4500	3.118	2.635	0.6769	0.6105	1.265	1.904	0.581	0.251
518	5400	2.899	2.395	0.6279	0.5548	1.266	1.822	0.547	0.219
519	6300	2.687	2.227	0.5802	0.5270	1.177	1.743	0.525	0.189
520	7200	2.490	1.851	0.5361	0.4289	1.358	1.670	0.400	0.156
521	9000	2.131	1.801	0.4749	0.4173	1.738	1.568	0.312	0.105
522	10800	1.842	1.638	0.3907	0.3794	1.0311	1.428	0.256	0.075
523	14400	1.414	1.036	0.2948	0.2399	1.169	1.269	0.159	0.058
524	18000	1.096	0.770	0.2234	0.1785	1.150	1.150	0.130	0.052
525	216000	0.860	0.567	0.1704	0.1314	1.147	1.063	0.108	0.050
526	252000	0.688	0.394	0.1319	0.0912	1.182	1.000	0.090	0.052
527	288000	0.576	0.274	0.1069	0.0636	1.232	0.957	0.064	0.049
528	360000	0.405	0.176	0.0683	0.0408	1.193	0.893	0.041	0.043
529	432000	0.323	0.106	0.0500	0.0245	1.238	0.863	0.034	0.053
530	504000	0.252	0.075	0.0342	0.0174	1.200	0.837	0.031	0.060
531	576000	0.215	0.047	0.0258	0.0124	1.201	0.823	0.027	0.080
532	648000	0.185	0.028	0.0191	0.0064	1.276	0.811	0.026	0.115
533	720000	0.175	0.019	0.0169	0.0045	1.327	0.808	0.025	0.151
534	792000	0.157	0.025	0.0129	0.0058	1.182	0.801	0.025	0.116

535 The matrix $[\hat{\alpha}]$ is a column matrix which elements are equal to the
 536 $(p - 1) = 24$ estimated values of α corresponding to the 25 measurements
 537 of m_i .

538 Three methods have been successively applied to estimate α : The least
 539 squares method, the simplified Gauss–Markov method suited if the errors
 540 on the α_i values are not correlated and the complete Gauss–Markov method
 541 suited even if the errors on the α_i values are correlated, these three methods
 542 leading respectively to the estimated values α_{ls} , α_{gmnc} and α_{gmc} .

543 The least squares method leads to the estimation of α as follows:

$$544 \alpha_{ls} = \frac{1}{p-1} \sum_{i=2}^p \alpha_i \quad \text{where} \quad \alpha_i = \frac{\ln(V_{r_i})}{\ln(X_{r_i})}$$

546

547 The simplified Gauss–Markov method leads to the calculation of α as
 548 follows:

549

$$550 \alpha_{\text{gmnc}} = \frac{\sum_{i=2}^p (\alpha_i / (d\alpha_i)^2)}{\sum_{i=2}^p (1 / (d\alpha_i)^2)}$$

551

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553 It can be deduced from Fig. 4 representing the experimental values of
 554 X_{r_i} , V_{r_i} , $\ln(X_{r_i})$ and α_i with their confidence interval how the same measurement
 555 error on each mass m_i leads to errors increasingly different
 556 between each measurement from X_{r_i} to α_i . So that it can already be forecast
 557 that even the simplified Gauss–Markov method will give better results than
 558 the least squares method since the experimental points having a great meas-
 559 urement uncertainty on α_i will have a very small influence on the finally
 560 estimated value of α .

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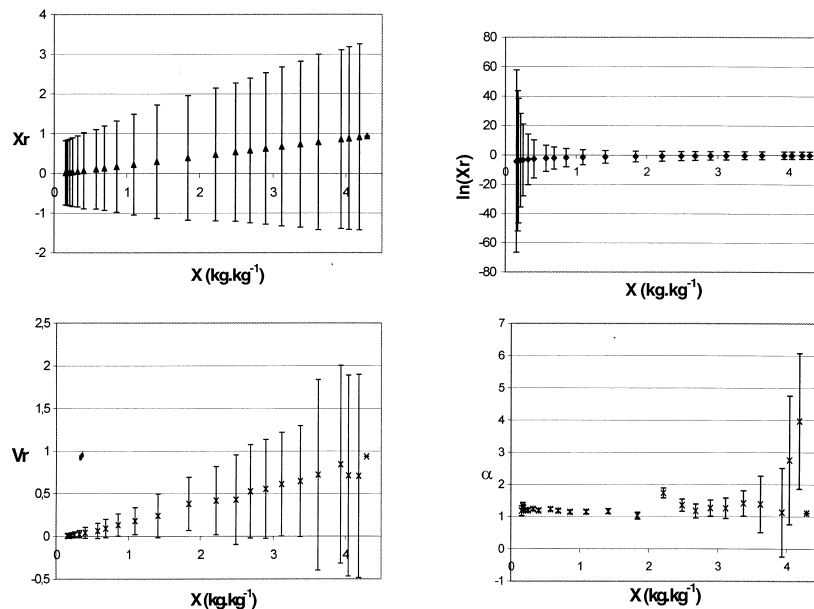
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587 **Figure 4.** Graphical representation of X_{r_i} , $\ln(X_{r_i})$, V_{r_i} and α_i with their confidence
 588 intervals.

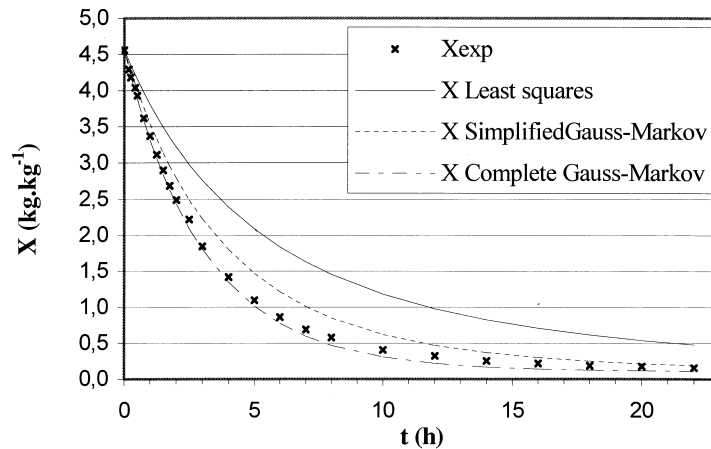


Figure 5. Experimental points and theoretical drying curves for a ginger drying experiment.

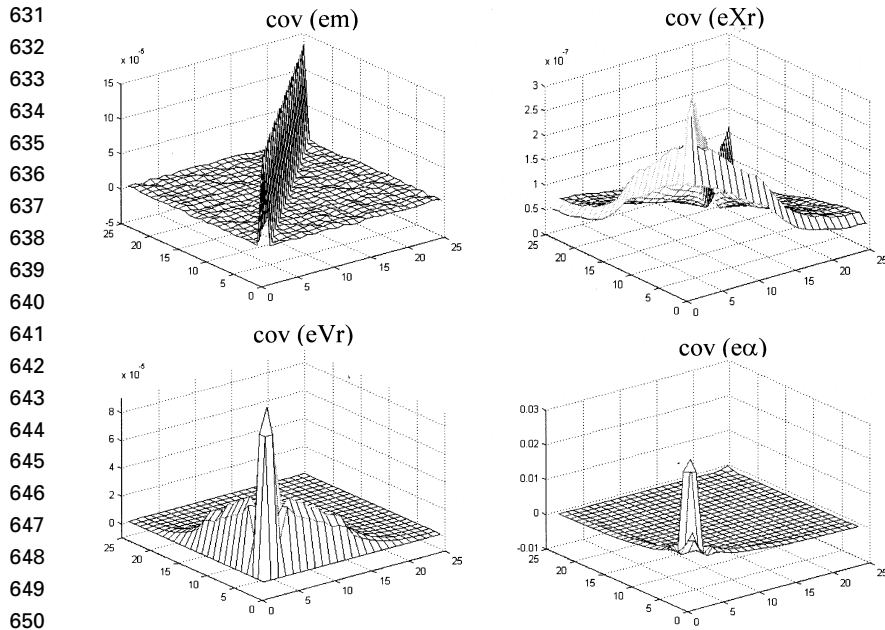
The graph $X=f(t)$ in Fig. 5 represents the 25 experimental points and the three theoretical curves calculated with the α values obtained with the three methods.

One can see that the theoretical curve using the α_{gmc} value calculated by the complete Gauss–Markov method (with the hypothesis that the errors are correlated) is quite close to all the experimental points. The theoretical curve using the α_{gmnc} value calculated by the simplified Gauss–Markov method (with the hypothesis that the errors are not correlated) is slightly distant from a few points in the middle of the curve but is quite close to the experimental points at the end of the drying. The gap between the theoretical curve using the α_{ls} value calculated by the ordinary least squares method and the experimental points is quite important all along the curve.

ERROR PROPAGATION AND SENSITIVITY ANALYSIS

The analysis of the errors covariance matrices of the various variables (graphically represented in Fig. 6) shows the following features:

- The covariance matrix is diagonal for non correlated variables such as m_i .
- The hypothesis that the matrix $[\text{cov}(e\alpha)]$ is diagonal is not justified in this case since all the $d\alpha_i$ values are linked to $d\alpha_1$ both through the reduced drying rate V_{r_i} which depends on V_1 and through



651 **Figure 6.** Graphical representation of the covariance matrices of the errors on m_i ,
652 X_{r_i} , V_{r_i} and α_i for the 25 experimental points.
653

654 X_{r_i} which depends on X_1 . The graphical representation of the
655 matrix $[\text{cov}(\alpha)]$ confirms this assertion.
656

657 It has been established that the product water content during drying is
658 given by (if the DCC is represented by a power function and if $\alpha \neq 1$):
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$$X(t) = X_{\text{eq}} + (X_{\text{cr}} - X_{\text{eq}}) \left[1 + \frac{(1 - \alpha)(V_1 t + X_1 - X_{\text{cr}})}{(X_{\text{cr}} - X_{\text{eq}})} \right]^{1/(1-\alpha)}$$

663 The sensitivity of each parameter to the calculated value of X is
664 deduced from the partial derivatives of X to these parameters. These deriva-
665 tives has been calculated as:
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$$\frac{\partial X}{\partial X_{\text{cr}}} = \left[1 + \frac{(1 - \alpha)(V_1 t + X_1 - X_{\text{cr}})}{X_{\text{cr}} - X_{\text{eq}}} \right]^{1/(1-\alpha)} + \left[1 + \frac{(1 - \alpha)(V_1 t + X_1 - X_{\text{cr}})}{X_{\text{cr}} - X_{\text{eq}}} \right]^{((1/1-\alpha)-1)} \frac{X_{\text{eq}} - V_1 t - X_1}{(X_{\text{cr}} - X_{\text{eq}})}$$

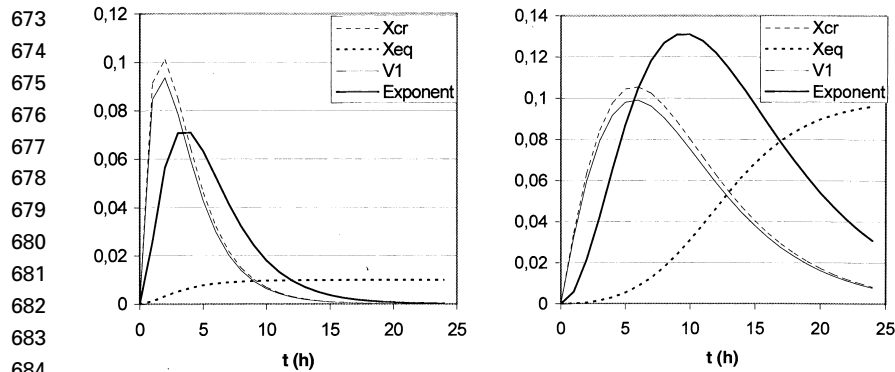


Figure 7. Graphical representation of the absolute and relative variation on X induced by a relative variation of 10% on the DCC parameters: X_{cr} , X_{eq} , V_1 , and α .

$$\begin{aligned} \frac{\partial X}{\partial X_{eq}} &= 1 - \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}} \right]^{1/(1-\alpha)} \\ &\quad + \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}} \right]^{((1/1-\alpha)-1)} \frac{(V_1 t + X_1 - X_{cr})}{(X_{cr} - X_{eq})} \\ \frac{\partial X}{\partial V_1} &= t \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}} \right]^{(1/1-\alpha)-1} \\ \frac{\partial X}{\partial \alpha} &= (X_{cr} - X_{eq}) \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}} \right]^{1/(1-\alpha)} \\ &\quad \times \left\{ \frac{1}{(1-\alpha)^2} \ln \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}} \right] \right. \\ &\quad \left. - \frac{1}{(1-\alpha) X_{cr} - X_{eq} + (1-\alpha)(V_1 t + X_1 - X_{cr})} \right\} \end{aligned}$$

The values of V_1 , X_{cr} , X_{eq} and α_{gmc} of the previous example related to ginger drying has been used to calculate and plot in Fig. 7 the evolution of:

F7

- $0.1 X_{cr} (\partial X / \partial X_{cr})$, $0.1 X_{eq} (\partial X / \partial X_{eq})$, $0.1 V_1 (\partial X / \partial V_1)$, and $0.1 \alpha (\partial X / \partial \alpha)$ as a function of X that represents the absolute variation of X when each of these parameters varies separately of 10% from their initial values.
- $0.1 (X_{cr}/X) (\partial X / \partial X_{cr})$, $0.1 (X_{eq}/X) (\partial X / \partial X_{eq})$, $0.1 (V_1/X) (\partial X / \partial V_1)$, and $0.1 (\alpha/X) (\partial X / \partial \alpha)$ as a function of X that represents the relative

715 variation of X when each of these parameters varies separately of
716 10% from their initial values.

717 It can be seen in Fig. 7 that variations on X_{eq} have an influence only at
718 the end of the drying (on the low values of X), that the most sensitive
719 parameter is α which remains sensitive at the end while variations on X_{cr}
720 and V_1 are influent at the beginning of the drying have a negligible influence
721 on the low values of X . A relative variation of 10% on X_{cr} or on V_1 leads to
722 an absolute variation of less than 0.01 kg kg^{-1} on the values of X at the end
723 of the drying that is low when compared with the X measurement uncertainty.
724 It can also be noted that variations on X_{cr} and V_1 have similar effects
725 on X that is a consequence of the relation:
726

$$727 \frac{dX}{dt} = \frac{V_1}{(X_{\text{cr}} - X_{\text{eq}})^\alpha} (X - X_{\text{eq}})^\alpha \quad \text{with: } X_{\text{cr}} - X_{\text{eq}} \approx X_{\text{cr}}$$

730 These sensitivity curves can be useful to analyse the residues of a
731 drying curve (differences between theoretical and experimental curves)
732 and specially to identify the parameter that must be primarily modified to
733 minimize these residues.
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739 CONCLUSIONS

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741 The results obtained for the identification of the exponent of the
742 drying characteristic curve of ginger underlines the interest of the
743 Gauss–Markov method for processing noisy experimental measurements
744 in some specific cases. In the case studied in this paper, a rather strong
745 correlation exists between the α_i . The complete Gauss–Markov method
746 (assuming that the errors are correlated) leads to quite significative
747 improvement compared with the ordinary least square method. The
748 improvement obtained in comparison with the simplified Gauss–Markov
749 method (assuming that the errors are not correlated) is less important.
750 Though the applicability conditions are not verified, this very easy to
751 apply method gives quite better results than the ordinary least squares
752 method. The use of the proposed estimation method may be helpful to
753 improve the choice of the time interval at which masses must be measured
754 to minimize the estimation error. The error propagation illustrated by the
755 graphical representation of the covariance matrices of errors shows that
756 measurement errors on masses are strongly amplified when one estimates
the DCC parameters. The sensitivity analysis gives information about the

COVARIANCE ANALYSIS OF DCC PARAMETERS

1937

757 importance and the variation along the drying curve of the sensibility of
 758 the DCC parameters, that could be useful to optimize the values of these
 759 parameters. The analysis of the residues (difference between experimental
 760 and «improved» theoretical curves) may also be useful to find mathemat-
 761 ical expressions different from the power function to represent the DCC if
 762 necessary.

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NOTATION

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768 a Constant769 b_i Constants770 B Matrix of constants b_i 771 dm_i Measurement uncertainties on mass m_i (kg)772 $d\alpha_i$ Estimation uncertainty on α_i induced by m_i measurement
 773 uncertainties774 em_{ij} Difference between the j th measured value of m_i and its
 775 exact value (kg)776 $e\alpha_{ij}$ Difference between the j th measured value α_i and its exact
 777 value

778 HR Relative humidity of the drying air (%)

779 m Mass of the fresh product set in the drier (kg)780 m' Mass of the fresh product used for X_1 determination (kg)781 m_d Bone dry mass of the product set in the drier (kg)782 m'_d Bone dry mass of the product used for X_1 determination
 783 (kg)784 M Mass of the product support in the drying apparatus (kg)785 t Drying time (s)786 v Drying air flow speed (m s^{-1})787 S Sensitivity matrix788 V Drying rate ($\text{kg kg}^{-1} \text{s}^{-1}$)789 V_r Reduced drying rate790 V_1 First phase drying rate ($\text{kg kg}^{-1} \text{s}^{-1}$)791 X_i Water content (kg kg^{-1})792 X_r Reduced water content793 X_1 Initial water content (kg kg^{-1})

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795 **Greek Symbols**

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797 α Exponent of the power function representing the drying
 798 characteristic curve

799 *Subscripts*

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801 i Variable value at time t_i 802 j Measurement number803 1 Variable value at time $t_1 = 0$

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