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MEASUREMENT ERRORS PROCESSING BY COVARIANCE ANALYSIS FOR AN IMPROVED ESTIMATION OF DRYING CHARACTERISTIC CURVE PARAMETERS

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ABSTRACT

The aim of this paper is to show the interest of the covariance analysis applied to measurement error in the particular case of the identification of a drying characteristic curve from experimental drying data. The modelisation of drying by use of the Drying Characteristic Curve (DCC) method is first presented with usual specifications (power function, critical moisture content...). The experimental procedure used to obtain drying curves and the data processing are detailled and analysed. Measurements errors are identified at the first step of the

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procedure and their effects on the estimation error of the exponent α of the power function are estimated. Three different methods for estimating α are presented under their matrix form: the least square method and two methods based on the ⟪Gauss–Markov⟫ or ⟪Maximum likelihood⟫ theorem, firstly under a simplified form suited if the estimation errors are uncorrelated and secondly under a complete form suited even if the estimation errors are correlated. These three methods are applied to experimental results obtained with ginger roots drying. The value of the exponent α of the power function and then the distances between the three corresponding theoretical drying curves (representing product water content vs. time) and the experimental points are studied. It is shown that in this particular application, the complete Gauss–Markov method leads to the better fitting and that the simplified Gauss–Markov method, since it is a priori non appliable in this case where errors are correlated, gives quite better results than the oridnary least squares method. The covariance matrices of the estimation errors of reduced water content, reduced drying rate and exponent α are also presented in order to show the correlations existing between the measurement errors of each variable during a drying cycle.

Key Words: Drying characteristic curve; Inverse method; Covariance analysis; Measurement error processing

INTRODUCTION

The ⟪Drying Characteristic Curve⟫ (DCC) is a concept proposed by Van Meel[1] in 1957. It consists in processing and reducing drying experimental results. The aim of such a processing is to obtain a single drying curve for a given product with given sizes, independently of the drying air conditions (temperature, humidity, speed), by use of convenient variables reductions applied to the product water content and to the drying rate.

Following Van Meel, many authors among them Desmorieux and Moyne,[2] Belahmidi et al.[3] Fornell et al.[4] Salgado et al.[5] and Ahouannou et al.,[6] have used this concept to characterize product properties with regard to drying.

Nevertheless, such authors do not describe deeply the way they take into account the measurement errors related to parameters estimation. Therefore, the aim of this paper is to show the interest of processing measurement errors by covariance analysis that can include the following aspects:

– Improved estimation of the DCC parameters.
– The study of the error propagation and sensitivity to DCC parameters can give information on the way to obtain the most confident result.

In this paper, the procedure to obtain the Drying Characteristic Curve (DCC) is first presented and analysed. The experimental procedure used to obtain drying curves and the mathematical calculation leading to the DCC from them are detailed. Error measurements at each step of the procedure and their effects on the estimation error of the exponent α of the power function are then described. Three different methods for estimating α are presented under their matrix form: the least square method and two methods based on the Gauss–Markov theorem. These three methods are applied to experimental results related to ginger roots drying in order to determine the value of the exponent α. The error propagation is analysed by the study of the covariance matrices of the estimation errors of reduced water content, reduced drying rate and exponent α. A sensitivity analysis of the different parameters is finally proposed.

THE DRYING CHARACTERISTIC CURVE METHOD

The ⟨⟨Drying Characteristic Curve⟩⟩ represents the reduced drying rate V_r as a function of the reduced water content X_r. These two variables are defined as follows:

$$V_r(t) = \frac{V(t)}{V_1} \quad \text{and} \quad X_r(t) = \frac{X(t) - X_{\text{eq}}}{X_{\text{cr}} - X_{\text{eq}}} \quad \text{with } V = \frac{dX}{dt}$$

$X(t)$: Dry basis product water content at time t
X_{eq}: Dry basis product water content when equilibrium between air and product is reached
X_{cr}: Dry basis product water content at the end of the first drying phase (initial phase with constant drying rate)
$V(t)$: Drying rate of the product at time t
V_1: Drying rate of the product during the first drying phase

A mathematical expression of the DCC for a product with fixed initial sizes is sought by analyzing the experimental drying curves obtained with various drying air temperature, humidity and speed conditions. One representative example of a presentation such as $dX/dt = f(X)$ is shown in Fig. 1. F1

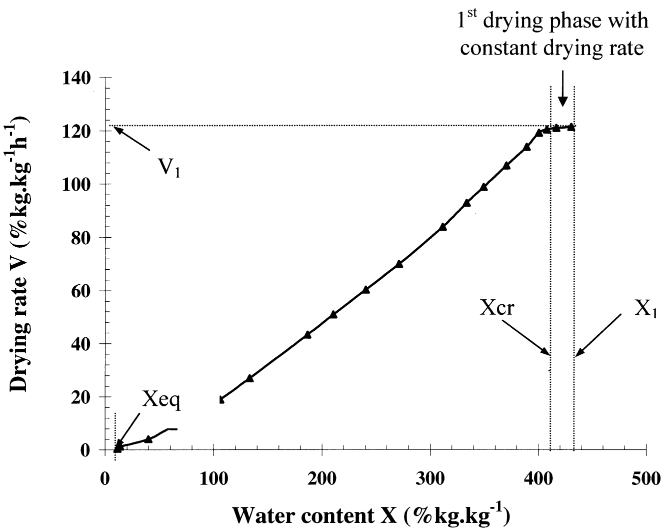

Figure 1. Example of drying curve $V = f(X)$ for mango with drying air at $T = 40°\mathrm{C}$, $HR = 15\%$ and $v = 1\,\mathrm{m\,s^{-1}}$.

For biological products, the raising temperature phase is most often negligible, especially if the difference between air and product temperatures is low and if the product dimensions are small. Our experimental results have confirmed this hypothesis.

The experimental curves $X_{(t)}$ have been derived to obtain the estimated curves $V(t) = dX/dt(t)$. Such analysis gives a mean value of the critical water content X_{cr} as shown in Fig. 1. Several authors (among them Desmorieux and Moyne[2]) have considered that for biological products it is difficult to identify a critical water content different from the initial water content X_1. Nevertheless, Talla et al.[7] could identify a critical water content $X_{cr} \neq X_1$ for banana and mango.

Then, from each experimental point (t, X), the couples of corresponding values (X_r, V_r) are calculated and the whole points obtained for various drying air conditions are placed on a unique graph representing $V_r = f(X_r)$ to obtain a cloud of points illustrated in Fig. 2. F2

A mathematical expression of the DCC: $V_r = f(X_r)$ which fit this cloud of points more or less dispersed is then sought. The function f having to respect the following conditions:

- If $X = X_{eq}$: $X_r = 0$ and $V = 0$ so $V_r = 0$ and finally $f(0) = 0$
- If $X = X_{cr}$: $X_r = 1$ and $V = V_1$ so $V_r = 1$ and finally $f(1) = 1$
- $0 < f(X) < 1$ if $0 < X_r < 1$.

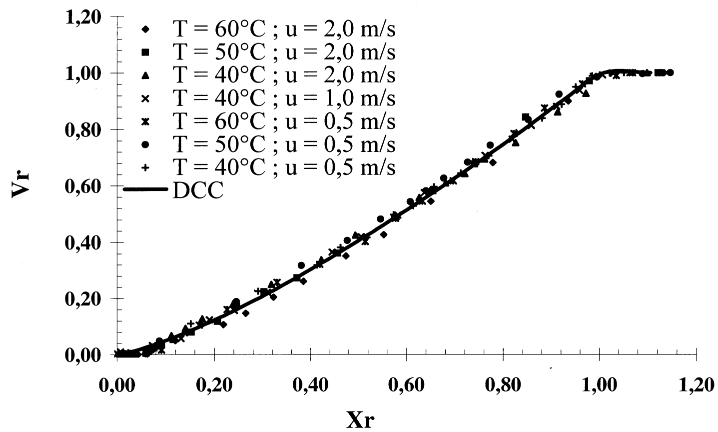

Figure 2. Drying characteristic curve for mango.

The function f is more often sought as a power function (Ahouannou et al.[6] and Talla et al.[7]) It also could be seek as a polynomial expression as noted by Desmorieux and Moyne.[2] In this study, the case of a power function $V_r = X_r^{\alpha}$ has been considered. This function is written in a developed form as:

$$\frac{V(t)}{V_1} = \left[\frac{X(t) - X_{eq}}{X_{cr} - X_{eq}}\right]^{\alpha} \qquad (1)$$

Experiments have been conducted within the drying tunnel depicted in Fig. 3, the objectives were: F3

- Obtaining experimental drying curves $X = f(t)$ for given products and air conditions
- Determining of the value of the exponent α which ensure the best fitting between the experimental points $X_{(t)}$ and the theoretical curves obtained by integration of Eq. (1):

$$\text{If } t \leq t_c \quad \text{where} \quad t_c = \frac{X_{cr} - X_1}{V_1} : X(t) = X_1 + V_1 t$$

$$\text{If } t > t_c \quad \text{if} \quad \alpha \neq 1:$$

$$X(t) = X_{eq} + (X_{cr} - X_{eq})\left[1 + \frac{(1-\alpha)V_1(t - t_c)}{(X_{cr} - X_{eq})}\right]^{1/(1-\alpha)}$$

$$\text{if } \alpha = 1: X(t) = X_{eq} + (X_{cr} - X_{eq}) \exp\left[\frac{V_1(t - t_c)}{X_{cr} - X_{eq}}\right]$$

where: t_c First drying time duration.

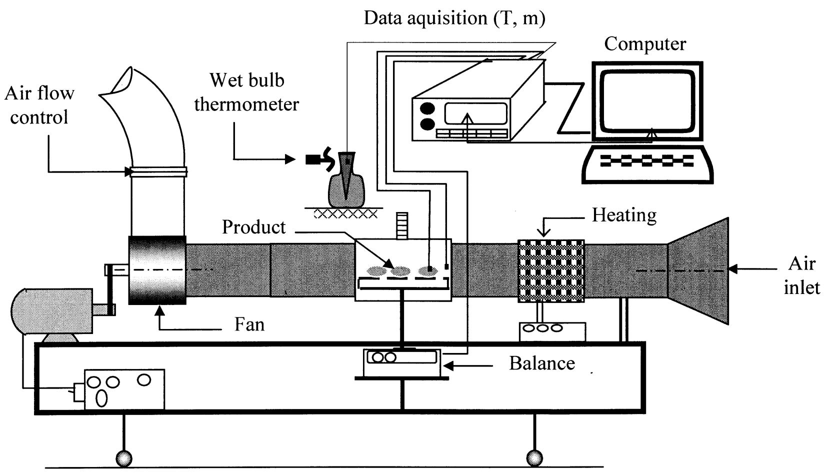

Figure 3. Schematic view of the drying tunnel.

EXPERIMENTAL DETERMINATION OF THE VARIOUS VARIABLES

It is considered here that the result $\hat{X}$ of a measurement or estimation of a parameter X is the sum of the true or exact value of X plus a random variable eX: $\hat{X} = X + eX$. The random variable eX is called the measurement error, the average of the k values eX_i $(i = 1, \ldots, k)$ obtained by repeating k times the measurement of X is considered to be equal to zero. What is usually called measurement uncertainty is a value dX such as the absolute values of eX_i are always lower than dX. So that dX can also be seen as the maximum observable value of the error eX. If the value $\hat{X}$ of the parameter X is directly obtained by use of a measuring apparatus, dX is called the precision of the apparatus.

Product Initial Water Content X_1 Determination

The fresh product (before any drying) water content is calculated as:

$$X_1 = \frac{m' - m'_d}{m'_d}$$

where m' is the mass of a piece of fresh product representative of the product to be dried and m'_d its bone dry mass. Both masses are estimated by difference between the mass of the product + support set and the mass of the support, this last one being measured before disposing the piece of fresh product on it. So, the measurement uncertainties dm' and dm'_d on the masses m' and m'_d obtained by the difference of two weighed masses is twice the precision Δm of the balance.

Product Water Content $X(t)$ Determination

A mass m_1 of fresh product is then placed on a support which mass M is determined by weighting. The product + support set mass $(M+m)_1$ is measured before introduction in the drying tunnel. The temperature, humidity and speed of the air flow are regulated in order to obtain constant conditions. The mass of the support + product set is measured at various times t_i until the product water content reached the desired final value. The product water content at time t_i is calculated as:

$$X_i = \frac{m_i - m_d}{m_d}$$

where:

$$m_i = (M+m)_i - M$$
$$m_d = \frac{m_1}{(1+X_1)}$$

The measurement uncertainties dm_i and dm_1 on the masses m_i and m_1 obtained by the difference of two weighed masses is twice the precision Δm of the balance.

Drying Rate $V(t)$ Determination

The drying rate V_i is deduced from the experimental points (t_i, X_i) using classical finite difference expressions:

– First Point $(t=0)$:

$$V_1 = \frac{X_1 - X_2}{t_2}$$

– Last point $(t=t_p)$:

$$V_p = \frac{X_{p-1} - X_p}{t_p - t_{p-1}}$$

– Intermediary point ($t = t_i$):

$$V_i = \frac{1}{2}\left(\frac{X_{i-1} - X_i}{t_i - t_{i-1}} + \frac{X_i - X_{i+1}}{t_{i+1} - t_i}\right)$$

Exponent α Determination

For each measurement at each time t_i, α_i is evaluated by use of the relation:

$$\alpha_i = \frac{\ln(V_{r_i})}{\ln(X_{r_i})} \quad \text{with } V_r = \frac{V_i}{V_1} \quad \text{and} \quad X_r = \frac{X_i - X_{\text{eq}}}{X_{\text{cr}} - X_{\text{eq}}}$$

MEASUREMENT OR ESTIMATION ERRORS

The following hypotheses have been considered:

- The measurement errors of times t_i at which measurements are realised are negligible.
- The estimation error on the evaluated values of X_{eq} and X_{cr} are negligible. This hypothesis concerning X_{cr} will be further justified.
- All the mass measurements are done with the same balance which precision is Δm.

The error calculation for the various variables leads to the following formula:

$$eX_1 = \frac{1}{m'_d}\left[em' + (X_1 - 1)em'_d\right]$$

$$eX_i = (X_i + 1)\left(\frac{em_i}{m_i} + \frac{em_1}{m_1} + \frac{eX_1}{1 + X_1}\right) \quad \text{for } i \neq 1$$

$$eX_{r_i} = X_{ri}\left(\frac{eX_i}{X_i - X_{\text{eq}}} + \frac{eX_1}{X_1 - X_{\text{eq}}}\right)$$

$$eV_1 = \frac{eX_1 - eX_2}{t_2 - t_1}$$

$$eV_p = \frac{eX_{p-1} - eX_p}{t_p - t_{p-1}}$$

$$eV_i = \frac{1}{2}\left(\frac{eX_{i-1} - eX_i}{t_i - t_{i-1}} + \frac{eX_i - eX_{i+1}}{t_{i+1} - t_i}\right)$$

$$eV_{r_i} = V_{r_i}\left(\frac{eV_1}{V_1} + \frac{eV_i}{V_i}\right)$$

$$e\alpha_i = \frac{eV_{r_i}}{V_{r_i}\ln(V_{r_i})} + \frac{\ln(V_{r_i})eX_{r_i}}{\left[\ln(X_{r_i})\right]^2}$$

The measurement uncertainties can be calculated by this formulas by taking as error measurement on the masses their uncertainties measurement that is $2\Delta m$, where Δm is the precision of the balance.

EXPERIMENTAL DATA ANALYSIS METHOD

A number p of measured values $\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_p$ acquired at times $t_1, t_2, \ldots, t_p$ are used to calculate $(p-1)$ corresponding values $\hat{\alpha}_2, \hat{\alpha}_3, \ldots, \hat{\alpha}_p$ of the researched exponent α. Uncertainties $d\alpha_i$ of these estimated values resulting from measurement uncertainties dm_i of m_i are also calculated by the previous formula with $\mathrm{em}' = em'_d = em_1 = em_i = 2\Delta m$. Uncertainty measurement dX_1 of X_1 is calculated in the same way.

A polynomial relationship is the most usual form sought in physical problems, in this case:

$$\alpha = b_n t^n + b_{n-1} t^{n-1} + \cdots + b_1 t + b_0$$

The relation between the exact value α_i of α at time t_i can be written under the form:

$$\begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} 1 & t_2 & \cdots & t_2^n \\ 1 & t_3 & \cdots & t_3^n \\ \vdots & \vdots & & \vdots \\ 1 & t_p & \cdots & t_p^n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix}$$

Or under a matrix form:

$$[\alpha] = [S][B] \text{ where } [S] = \left[\left(\frac{\partial f}{\partial b_i} \right)_{t_j} \right] \text{ is the sensitivity matrix.}$$

The determination of the matrix $[B]$ when the matrix $[S]$ is known without error and the matrix $[\hat{\alpha}]$ is known with unnegligible errors can be done using one of the two followings methods:

The Ordinary Least Square Method

This classical method as described by Trigeassou[8] and Press et al.[9] among many authors can be applied to calculate the vector $[B]$ which minimise the quadratic error D between the measures values vector $[\hat{\alpha}]$ and the exact values vector $[\alpha]$, D being calculated as follows:

$$D = \left\|[\hat{\alpha}] - [S][B]\right\| = ([\hat{\alpha}] - [S][B])^t([\hat{\alpha}] - [S][B])$$

The vector B which minimise D is given by:

$$[B] = ([S]^t[S])^{-1}S^t[\hat{\alpha}]$$

Gauss–Markov Method

According to Beck and Arnold,[10] if the following conditions are verified:

- The matrix $[S]$ is known without error,
- The errors on α_i have zero mean,
- The covariance matrix of measurement error is known with exception of a multiplicative constant a, so that a matrix P is known that verifies $[P] = a\,[\mathrm{cov}(e\alpha)]$,
- The matrix $[P]$ is positive definite,

then the vector minimising the difference:

$$D = ([\hat{\alpha}] - [S][B])^t[\mathrm{cov}(e\alpha)]^{-1}([\hat{\alpha}] - [S][B])$$

is given by:

$$[B] = ([S]^t[P]^{-1}[S])^{-1}[S]^t[P]^{-1}[\hat{\alpha}]$$

This evaluation method called "Gauss–Markov method" take into account the differences between the measurement errors of the various α_i values and minimise the importance of the α_i values with a great measurement error. The least square method considers implicitely that all the α_i values have the same measurement errors.

The most important problem linked to the application of this method is the evaluation of the matrix $[\mathrm{cov}(e\alpha)]$. Two cases may occur:

Simplified method: the measures of α are not correlated i.e., the measurement error of α_i is independent of the measurement error of α_j. In this specific case: $\mathrm{cov}(e\alpha_i, e\alpha_j) = 0$ if $i \neq j$ and the matrix $[\mathrm{cov}(e\alpha)]$ is diagonal:

$$[\mathrm{cov}(e\alpha)] = \begin{bmatrix} \mathrm{var}(e\alpha_1) & & & \\ & \mathrm{var}(e\alpha_2) & & 0 \\ & & \ddots & \\ 0 & & & \mathrm{var}(e\alpha_p) \end{bmatrix}$$

Whether the hypothesis that α_i measurement is errorless, the mean value $E(e\alpha_{ij})$ at i constant of the measurement errors $(e\alpha_{ij})$ is equal to zero and $\mathrm{var}(e\alpha_i)$ may be written as:

$$\mathrm{var}(e\alpha_i) = \frac{1}{k}\sum_{j=1}^{k}[e\alpha_{ij} - E(e\alpha_{ij})]^2 = \frac{1}{k}\sum_{j=1}^{k}(e\alpha_{ij})^2 = \frac{1}{k}\sum_{j=1}^{k}(r_j d\alpha_i)^2$$

$$= (d\alpha_i)^2 \frac{1}{k}\sum_{j=1}^{k} r_j^2$$

where r_j is a random number bounded by -1 and $+1$.

With the hypothesis that all the measured values α_{ij} of α at time t_i are dispersed in the same manner around the mean value α_i, it may be written:

$$\frac{1}{k}\sum_{j=1}^{k} r_j^2 = a$$

and the covariance matrix take the following form:

$$[\mathrm{cov}(e\alpha)] = a\begin{bmatrix} (d\alpha_1)^2 & & \\ & (d\alpha_2)^2 & 0 \\ 0 & & \ddots \\ & & & (d\alpha_p)^2 \end{bmatrix}$$

Complete method: The measured values of α are correlated i.e., measurement error of α_i is related to measurement error of α_j, so that the previous method cannot be applied. Nevertheless, the information contained in measurement uncertainty value dm_i and in the errors calculus formula may be used in the following way:

k measurements of $X_1, m_1, m_2, \ldots, m_p$ are simulated by giving the value $\hat{X}_1 = X_1 + r_j dX_1$ to the jth simulated value of X_1 and the value $\hat{m}_{ij} = m_i + r_{ij} dm_i$ to the jth simulated value of m_i, where r_j and r_{ij} are random numbers bounded by -1 and $+1$. Using the previously established errors formula, the values $X_{ij}, V_{ij}, X_{r_{ij}}, V_{r_{ij}}, \alpha_{ij}$ corresponding to the masses $\hat{m}_{ij}$ and the errors eX_{ij}, eV_{ij}, $eX_{r_{ij}}, eV_{r_{ij}}$ and $e\alpha_{ij}$ induced by the error em_{ij} are evaluated for i varying from 1 to p and for j varying from 1 to k. A matrix $[e\alpha]$ of the simulated values $e\alpha_{ij}$ is thus obtained which enables an evaluation of the matrix $[\mathrm{cov}(e\alpha)]$ and the application of the Gauss–Markov method. The above described methods may be applied to relations other than polynomial on condition that this relation be linear toward b_i that is to say $(\partial/\partial b_i)[\partial f(t)/\partial b_j] = 0$, for all values of i varying from 1 to p and of j varying from 1 to k (the sensibility matrix S must not depend on b_i).

Method Comparison

The previously described methods have been applied for the processing of experimental data of a drying cycle: the results of a drying experiment of ginger roots cut in 50 mm long pieces and disposed in an air flowing at $T = 40°C$, $v = 1$ m/s and HR = 32%. A first drying phase with a constant drying rate could not be identified so the critical water content X_{cr} is considered as equal to the initial water content X_1. The equilibrium water content X_{eq} has been calculated by applying Ahouannou et al.[6] results that leads to the value $X_{eq} = 0.1\ \mathrm{kg\,kg^{-1}}$. A balance with a precision of 0.01 g has been used for weight measurements, the experimental results are presented in Table 1. T1

The initial water content (dry basis) value of $X_1 = 4.559$ kg/kg has been obtained by complete dehydration of a separate sample (representative of the product to be dried) with a final bone dry mass $m_d = 30$ g. The calculation of the X_i, V_i, V_{r_i},X_{r_i}, α_i values and of their uncertainties leads to the results presented in Table 2, the measurement uncertainties of the various masses being equal to 0.02 g that is twice the balance precision. T2

The problem to be solved is the simplest one where a constant and time independent value α is sought which correspond to the estimation of $\alpha = f(t) = b_0$. In this case, the matrix [S] is a column matrix with p rows and each element being equal to 1:

$$S = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Table 1. Experimental Values of the Ginger Mass During Its Drying

t_i (h)	0	0.17	0.25	0.42	0.5	0.75	1	1.25	1.5
m_i (g)	46.30	44.11	43.19	42.02	41.09	38.48	36.43	34.30	32.48
t_i (h)	1.75	2	2.5	3	4	5	6	7	8
m_i (g)	30.71	29.07	26.08	23.67	20.11	17.46	15.49	14.06	13.13
t_i (h)	10	12	14	16	18	20	22		
m_i (g)	11.7	11.02	10.43	10.12	9.87	9.79	9.64		

Table 2. Calculated Values of the Water Contents X and X_r, the Drying Rates V and V_r, the Exponent α and Their Uncertainties During the Drying

t_i (s)	X_i ($kg\,kg^{-1}$)	$10^4\ V_i$ ($kg\,kg^{-1}\,s^{-1}$)	X_{r_i}	V_{r_i}	α_i	$10^3\ dX_{r_i}$	$10^{-1}\ dV_{r_i}$	$d\alpha_i$
0	4.559	4.296	1	1	—	0	0	—
612	4.296	4.066	0.9408	0.9347	1.107	2.342	1.193	2.111
900	4.186	3.065	0.9165	0.7074	3.972	2.301	1.179	1.999
1512	4.045	3.086	0.8842	0.7122	2.758	2.249	1.161	1.376
1800	3.933	3.679	0.8597	0.8429	1.131	2.207	1.118	0.880
2700	3.620	3.108	0.7894	0.7202	1.388	2.091	0.651	0.396
3600	3.374	2.788	0.7342	0.6460	1.414	2.000	0.608	0.315
4500	3.118	2.635	0.6769	0.6105	1.265	1.904	0.581	0.251
5400	2.899	2.395	0.6279	0.5548	1.266	1.822	0.547	0.219
6300	2.687	2.227	0.5802	0.5270	1.177	1.743	0.525	0.189
7200	2.490	1.851	0.5361	0.4289	1.358	1.670	0.400	0.156
9000	2.131	1.801	0.4749	0.4173	1.738	1.568	0.312	0.105
10800	1.842	1.638	0.3907	0.3794	1.0311	1.428	0.256	0.075
14400	1.414	1.036	0.2948	0.2399	1.169	1.269	0.159	0.058
18000	1.096	0.770	0.2234	0.1785	1.150	1.150	0.130	0.052
216000	0.860	0.567	0.1704	0.1314	1.147	1.063	0.108	0.050
252000	0.688	0.394	0.1319	0.0912	1.182	1.000	0.090	0.052
288000	0.576	0.274	0.1069	0.0636	1.232	0.957	0.064	0.049
360000	0.405	0.176	0.0683	0.0408	1.193	0.893	0.041	0.043
432000	0.323	0.106	0.0500	0.0245	1.238	0.863	0.034	0.053
504000	0.252	0.075	0.0342	0.0174	1.200	0.837	0.031	0.060
576000	0.215	0.047	0.0258	0.0124	1.201	0.823	0.027	0.080
648000	0.185	0.028	0.0191	0.0064	1.276	0.811	0.026	0.115
720000	0.175	0.019	0.0169	0.0045	1.327	0.808	0.025	0.151
792000	0.157	0.025	0.0129	0.0058	1.182	0.801	0.025	0.116

The matrix $[\hat{\alpha}]$ is a column matrix which elements are equal to the $(p-1) = 24$ estimated values of α corresponding to the 25 measurements of m_i.

Three methods have been successively applied to estimate α: The least squares method, the simplified Gauss–Markov method suited if the errors on the α_i values are not correlated and the complete Gauss–Markov method suited even if the errors on the α_i values are correlated, these three methods leading respectively to the estimated values α_{ls}, α_{gmnc} and α_{gmc}.

The least squares method leads to the estimation of α as follows:

$$\alpha_{ls} = \frac{1}{p-1}\sum_{i=2}^{p}\alpha_i \quad \text{where} \quad \alpha_i = \frac{\ln(V_{r_i})}{\ln(X_{r_i})}$$

The simplified Gauss–Markov method leads to the calculation of α as follows:

$$\alpha_{\text{gmnc}} = \frac{\sum_{i=2}^{p} (\alpha_i/(d\alpha_i)^2)}{\sum_{i=2}^{p} (1/(d\alpha_i)^2)}$$

It can be deduced from Fig. 4 representing the experimental values of X_{r_i}, V_{r_i}, $\ln(X_{r_i})$ and α_i with their confidence interval how the same measurement error on each mass m_i leads to errors increasingly different between each measurement from X_{r_i} to α_i. So that it can already be forecast that even the simplified Gauss–Markov method will give better results than the least squares method since the experimental points having a great measurement uncertainty on α_i will have a very small influence on the finally estimated value of α. F4

For the complete Gauss–Markov method, a number of $k = 10{,}000$ measurements have been simulated.

The following values have been estimated: $\alpha_{\text{ls}} = 1.401$, $\alpha_{\text{gmnc}} = 1.185$ and $\alpha_{\text{gmc}} = 1.041$.

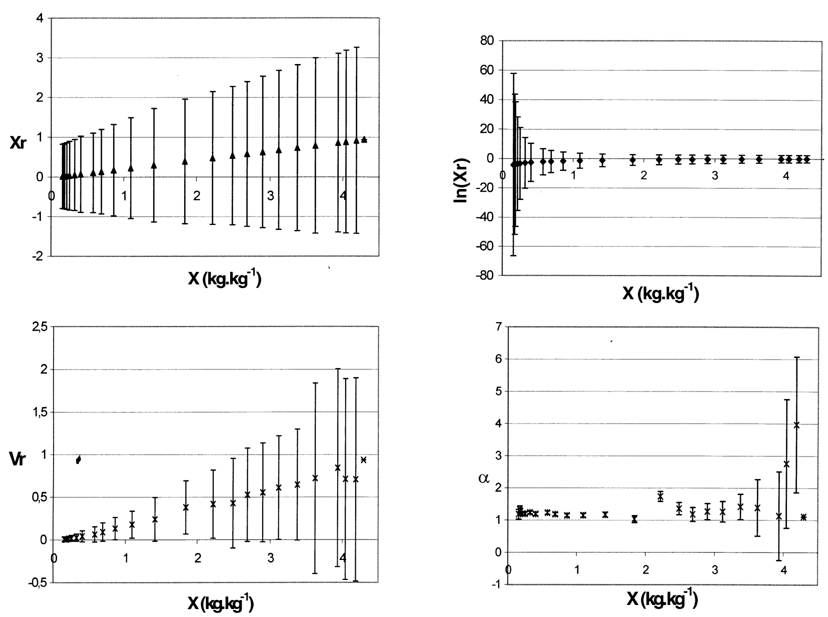

Figure 4. Graphical representation of X_{r_i}, $\ln(X_{r_i})$, V_{r_i} and α_i with their confidence intervals.

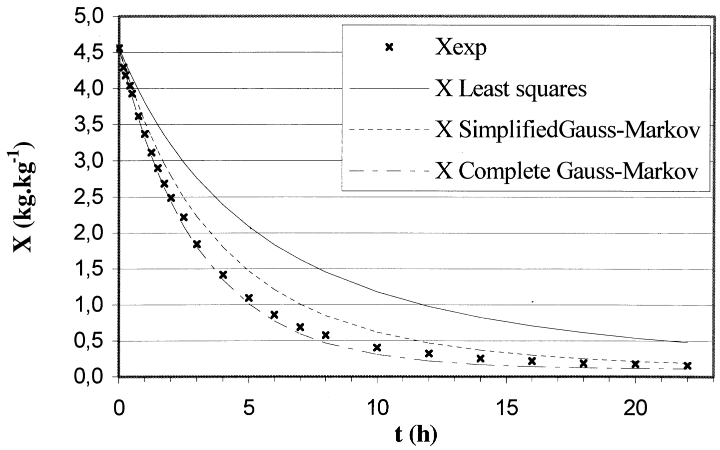

Figure 5. Experimental points and theoretical drying curves for a ginger drying experiment.

The graph $X = f(t)$ in Fig. 5 represents the 25 experimental points and the three theoretical curves calculated with the α values obtained with the three methods. F5

One can see that the theoretical curve using the α_{gmc} value calculated by the complete Gauss–Markov method (with the hypothesis that the errors are correlated) is quite close to all the experimental points. The theoretical curve using the α_{gmnc} value calculated by the simplified Gauss–Markov method (with the hypothesis that the errors are not correlated) is slightly distant from a few points in the middle of the curve but is quite close to the experimental points at the end of the drying. The gap between the theoretical curve using the α_{ls} value calculated by the ordinary least squares method and the experimental points is quite important all along the curve.

ERROR PROPAGATION AND SENSITIVITY ANALYSIS

The analysis of the errors covariance matrices of the various variables (graphically represented in Fig. 6) shows the following features: F6

- The covariance matrix is diagonal for non correlated variables such as m_i.
- The hypothesis that the matrix $[\mathrm{cov}(e\alpha)]$ is diagonal is not justified in this case since all the $d\alpha_i$ values are linked to $d\alpha_1$ both through the reduced drying rate V_{r_i} which depends on V_1 and through

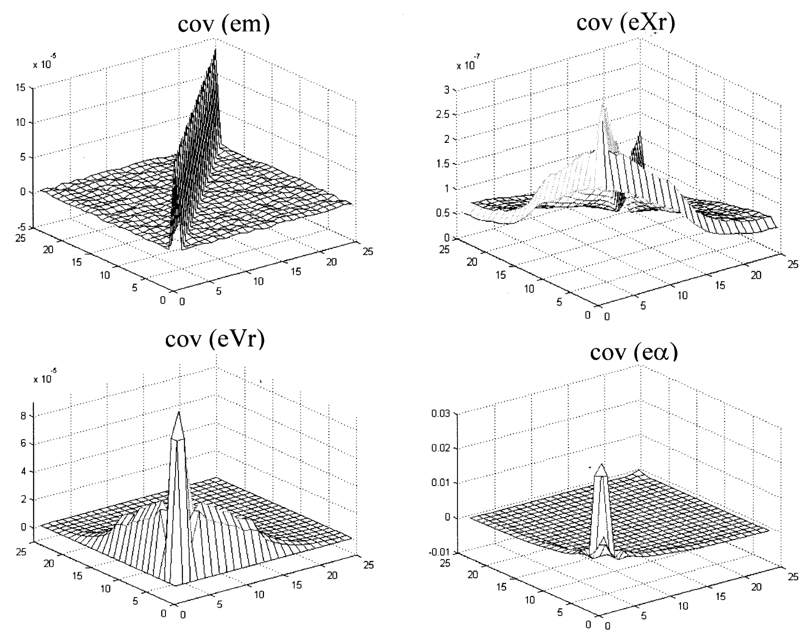

Figure 6. Graphical representation of the covariance matrices of the errors on m_i, X_{r_i}, V_{r_i} and α_i for the 25 experimental points.

X_{r_i} which depends on X_1. The graphical representation of the matrix [cov(eα)] confirms this assertion.

It has been established that the product water content during drying is given by (if the DCC is represented by a power function and if $\alpha \neq 1$):

$$X(t) = X_{\mathrm{eq}} + (X_{\mathrm{cr}} - X_{\mathrm{eq}})\left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{\mathrm{cr}})}{(X_{\mathrm{cr}} - X_{\mathrm{eq}})}\right]^{1/(1-\alpha)}$$

The sensitivity of each parameter to the calculated value of X is deduced from the partial derivates of X to these parameters. These derivatives has been calculated as:

$$\frac{\partial X}{\partial X_{\mathrm{cr}}} = \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{\mathrm{cr}})}{X_{\mathrm{cr}} - X_{\mathrm{eq}}}\right]^{1/(1-\alpha)} + \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{\mathrm{cr}})}{X_{\mathrm{cr}} - X_{\mathrm{eq}}}\right]^{((1/1-a)-1)} \frac{X_{\mathrm{eq}} - V_1 t - X_1}{(X_{\mathrm{cr}} - X_{\mathrm{eq}})}$$

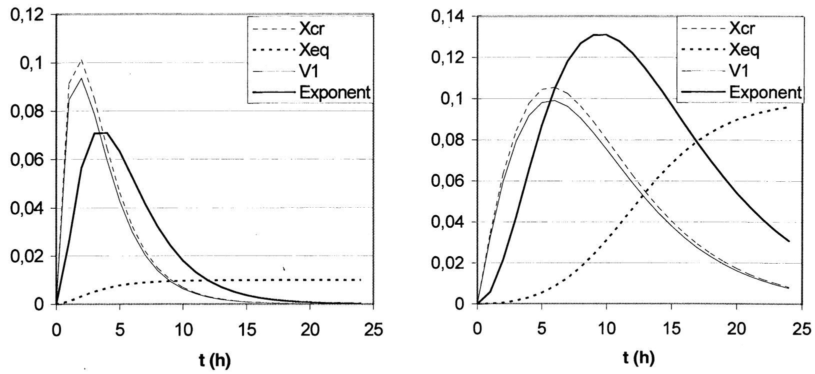

Figure 7. Graphical representation of the absolute and relative variation on X induced by a relative variation of 10% on the DCC parameters: X_{cr}, X_{eq}, V_1, and α.

$$\frac{\partial X}{\partial X_{eq}} = 1 - \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}}\right]^{1/(1-\alpha)} + \left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}}\right]^{((1/1-\alpha)-1)} \frac{(V_1 t + X_1 - X_{cr})}{(X_{cr} - X_{eq})}$$

$$\frac{\partial X}{\partial V_1} = t\left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}}\right]^{(1/1-\alpha)-1)}$$

$$\frac{\partial X}{\partial \alpha} = (X_{cr} - X_{eq})\left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}}\right]^{1/(1-\alpha)} \times \left\{\frac{1}{(1-\alpha)^2}\ln\left[1 + \frac{(1-\alpha)(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq}}\right] - \frac{1}{(1-\alpha)}\frac{(V_1 t + X_1 - X_{cr})}{X_{cr} - X_{eq} + (1-\alpha)(V_1 t + X_1 - X_{cr})}\right\}$$

The values of V_1, X_{cr}, X_{eq} and α_{gmc} of the previous example related to ginger drying has been used to calculate and plot in Fig. 7 the evolution of: F7

– $0.1X_{cr}(\partial X/\partial X_{cr})$, $0.1X_{eq}(\partial X/\partial X_{eq})$, $0.1V_1(\partial X/\partial V_1)$, and $0.1\alpha(\partial X/\partial \alpha)$ as a function of X that represents the absolute variation of X when each of these parameters varies separatly of 10% from their initial values.
– $0.1(X_{cr}/X)(\partial X/\partial X_{cr})$, $0.1(X_{eq}/X)(\partial X/\partial X_{eq})$, $0.1(V_1/X)(\partial X/\partial V_1)$, and $0.1(\alpha/X)(\partial X/\partial \alpha)$ as a function of X that represents the relative

variation of X when each of these parameters varies separatly of 10% from their initial values.

It can be seen in Fig. 7 that variations on X_{eq} have an influence only at the end of the drying (on the low values of X), that the most sensitive parameter is α which remains sensitive at the end while variations on X_{cr} and V_1 are influent at the beginning of the drying have a negligible influence on the low values of X. A relative variation of 10% on X_{cr} or on V_1 leads to an absolute variation of less that $0.01\,\mathrm{kg\,kg^{-1}}$ on the values of X at the end of the drying that is low when compared with the X measurement uncertainty. It can also be noted that variations on X_{cr} and V_1 have similar effects on X that is a consequence of the relation:

$$\frac{dX}{dt} = \frac{V_1}{(X_{cr} - X_{eq})^{\alpha}}(X - X_{eq})^{\alpha} \quad \text{with:} \quad X_{cr} - X_{eq} \approx X_{cr}$$

These sensitivity curves can be useful to analyse the residues of a drying curve (differences between theoretical and experimental curves) and specially to identify the parameter that must be primarily modified to minimize these residues.

CONCLUSIONS

The results obtained for the identification of the exponent of the drying characteristic curve of ginger underlines the interest of the Gauss–Markov method for processing noisy experimental measurements in some specific cases. In the case studied in this paper, a rather strong correlation exists between the α_i. The complete Gauss–Markov method (assuming that the errors are correlated) leads to quite significative improvement compared with the ordinary least square method. The improvement obtained in comparison with the simplified Gauss–Markov method (assuming that the errors are not correlated) is less important. Though the applicability conditions are not verified, this very easy to apply method gives quite better results than the ordinary least squares method. The use of the proposed estimation method may be helpful to improve the choice of the time interval at which masses must be measured to minimize the estimation error. The error propagation illustrated by the graphical representation of the covariance matrices of errors shows that mesurement errors on masses are strongly amplified when one estimates the DCC parameters. The sensitivity analysis gives information about the importance and the variation along the drying curve of the sensibility of the DCC parameters, that could be useful to optimize the values of these parameters. The analysis of the residues (difference between experimental and ⟨⟨improved⟩⟩ theoretical curves) may also be useful to find mathematical expressions different from the power function to represent the DCC if necessary.

NOTATION

a	Constant
b_i	Constants
B	Matrix of constants b_i
dm_i	Measurement uncertainties on mass m_i (kg)
$d\alpha_i$	Estimation uncertainty on α_i induced by m_i measurement uncertainties
em_{ij}	Difference between the jth measured value of m_i and its exact value (kg)
$e\alpha_{ij}$	Difference between the jth measured value α_i and its exact value
HR	Relative humidity of the drying air (%)
m	Mass of the fresh product set in the drier (kg)
m'	Mass of the fresh product used for X_1 determination (kg)
m_d	Bone dry mass of the product set in the drier (kg)
m'_d	Bone dry mass of the product used for X_I determination (kg)
M	Mass of the product support in the drying apparatus (kg)
t	Drying time (s)
v	Drying air flow speed ($m\,s^{-1}$)
S	Sensitivity matrix
V	Drying rate ($kg\,kg^{-1}\,s^{-1}$)
V_r	Reduced drying rate
V_1	First phase drying rate ($kg\,kg^{-1}\,s^{-1}$)
X_i	Water content ($kg\,kg^{-1}$)
X_r	Reduced water content
X_1	Initial water content ($kg\,kg^{-1}$)

Greek Symbols

α	Exponent of the power function representing the drying characteristic curve

Subscripts

i	Variable value at time t_i
j	Measurement number
1	Variable value at time $t_1 = 0$

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