Study of a simple transient non-intrusive sensor for internal temperature estimation during food product freezing

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Abstract

It is first recalled that the temperature profile in a freezing product is almost linear. To estimate the freezing level of a food product, a non-intrusive sensor is proposed which may estimate the surface temperature and the slope of the internal temperature profile. It consists in analysing transient response when the sensor is set in contact with the product to characterise. The sensor is made of a thermopile (Peltier element) fixed on an isothermal medium made of a high effusivity material (copper). A modelling of the transient response of the sensor using the quadrupoles method is presented. Then, the results of the characterisation of the sensor by experimental determination of the transfer functions between fluxes and temperatures are given. A theoretical sensitivity analysis enables to study of the potential limits of this sensor. Finally, the influence of internal temperature profile slope on the sensor response is evidenced by the results of an experimental study. The main interests of this sensor are its simplicity and low cost and a flux measurement independent of thermal contact resistance to estimate a temperature.

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1. Introduction

During the freezing of a product by a cold airflow, heat is evacuated from the product to the air by convection heat transfer. The product is gradually cooled from surface to centre. Its temperature decreases more slowly than in a cooling without phase change because of solidification latent heat that must be conducted out the product and then evacuated by convection at the surface. Since the freezing latent heat is very high, the motion of the freezing front (at 0 °C) is very slow. As shown by Carslaw and Jaeger [1], that leads to a quasi-stationary linear temperature profile in the frozen zone with possible high temperature gradient mainly at the beginning of the freezing. Such situation is very usual in the case of foodstuff storage at low temperature. This is
illustrated in Fig. 1, which represents the internal temperature profile in a piece of beef during its freezing, by an airflow at 7.6 m s$^{-1}$ and $-31.7^\circ$C (according to Moshenin [2]).

Fig. 2 (Moshenin [2]) indicates the difference between temperature at several depths and temperature at 0.5 cm from surface in a freezing piece of beef. It clearly shows great differences that cannot be detected by surface temperature measurements. It is thus necessary to get a method for estimating the temperature at a distance $x$ far from the surface to know whether a product is totally or superficially frozen.

Fig. 3 represents the evolution of temperature profile inside a piece of beef during its freezing (Moshenin). Taking into account the measurement errors, this profile could be considered as quasi-linear as predicted by the Carslaw and Jaeger [1] model so that the estimation of surface temperature and of the temperature profile slope could characterise the temperature field. The magnitude order of the slopes observable on this example is 400 K m$^{-1}$ after 6 h, 200 K m$^{-1}$ after 10 h and 100 K m$^{-1}$ after 14 h.

Intrusive temperature measurement inside a product is quite difficult to realise with precision when the temperature difference between the cooling air and the product heart is important (often 15–20 $^\circ$C), the thermal sensor is then like a cooling fin which causes temperature decrease around it and leads to temperature measured values inferior to true values according to Bardon [3]. The response time of the sensor is another error source as shown by Bourouga et al. [4]. As a result, the measured temperature indicated by an intrusive thermometer may be different by several degrees from the true value as shown by the results of Zehua Hu [5].

Furthermore, frozen meat is a very hard product in which it is quite impossible to use a penetrating thermal sensor to measure an interior temperature. The surface temperature measure is also very difficult to realise because the contact thermal resistance and the fin effect may cause non-negligible measurement errors according to Bardon [3].

These difficulties linked to interior and surface temperature measurements lead us to test a new measurement method: a thermal perturbation is created at the surface of the product by applying on it a thermal probe whose temperature at several depths and temperature at 0.5 cm from surface. It clearly shows great differences that cannot be detected by surface temperature measurements. It is thus necessary to get a method for estimating the temperature at a distance $x$ far from the surface to know whether a product is totally or superficially frozen.
transient temperature is then recorded. This transient temperature depends on internal temperature profile so that its analysis may lead to an estimation of the surface temperature and of the slope of the linear supposed temperature profile by use of an inverse method. Contrarily to Nguyen [6] who used a surface temperature measurement to estimate an internal temperature profile, the proposed method uses the signal of a probe in contact with the surface and an inverse method is applied to estimate both surface temperature, the slope (supposed constant) of the internal temperature profile and the contact thermal resistance between the probe and the product.

To satisfy user constraints, the aim is to design a low cost, non-intrusive and easy to use system. The proposed sensor is based on heat transfer analysis at the interface between the product to be characterised and an isothermal reference mass (a sample of copper). An explicative scheme of the measurement device is given in Fig. 4.

The idea is based on the analysis of the transient response of a Peltier element set at the interface between an isothermal mass and the product using a three-dimensional transient modelling by the quadrupole method (as developed by Maillet et al. [7]) and parameters estimation methods. It is expected that knowing the reference temperature $T_0$ (supposed constant) and the transient electrical tension $U_p(t)$ given by the thermopile it will be possible to estimate the unknown parameters of the product: surface temperature, slope of the internal temperature profile and contact thermal resistance between product and Peltier element.

The realisation of such a measurement needs several achievements:

- characterisation of the sensor (isothermal mass and Peltier element stuck on it): modelling and characteristic parameters estimation;
- heat transfer modelling during the measurement process and sensitivity study of the thermopile response to product parameters;
- determination of the thermal properties of the product on which a measurement is to be done;
- definition and test of an estimation method for

![Fig. 4. Measurement device.](image-url)
temperatures (surface and interior) and thermal contact resistance using experimental values of transient electrical tension $U_p(t)$ of the thermopile;

– experimental validation of the method.

2. Sensor description

The sensor is made of a Peltier element whose dimensions are $1.5 \times 1.5 \times 0.31$ cm$^3$ stuck with silver paint on a plane face of a copper made cylinder 6 cm in diameter and 6 cm in height.

The thermopile (made of two ceramic plates 1 mm in thickness) has been chosen to realise a thermal measurement between the product and the cylinder for the following reasons:

– it delivers an electrical tension of approximately 10 mV per degree difference between the two ceramic plates, so that its recording needs no signal amplifier;

– it is a low cost element.

The copper has been selected for the isothermal mass because of its high thermal effusivity. Its mean temperature may be considered as a constant when moderates flux passes through the Peltier element during a time around 60 s. Its high thermal conductivity may justify the hypothesis that the temperature is uniform in the cylinder.

The silver paint used to stick the thermopile on the copper cylinder is justified by its high thermal conductivity that reduces the thermal contact resistance.

The temperature (assumed to be uniform) of the cylinder is measured by a thermocouple set on the cylinder axis at a distance of 1 cm from the face on which the Peltier element is stuck.

A first experiment was realised for verifying the linearity of the relation between the sensor tension $U_p$ and the temperature difference $\Delta T$ between the two ceramic plates. This device is represented in Fig. 5: a thermocouple with separated contacts has been stuck on the external face of the thermopile. Then, a heating resistance with the same surface has been set on it and recovered with a thermal insulator (polyurethane foam) to avoid thermal losses.

A thermopile is constituted by two plates linked one to the other by thermocouples set in series so that it could be predicted that the electrical tension delivered by this element is proportional to the difference of the internal temperature of these two plates. An experimental verification has been carried out: flux steps of several values have been applied to the heating resistance and the stationary values of the electrical tension $U_p$ and of the temperature $T_0$ and $T_3$ have been recorded. The linearity of the relation between the stationary values of the tension $U_p$ and the difference of the external temperature of the two plates has been verified as shown in Fig. 6. These temperature values being stationary it can be deduced that the relation between the tension $U_p$ and the internal temperatures of the two plates is also linear.

3. Heat transfer modelling

The operating mode of the sensor previously described consists in setting this sensor initially at a uniform temperature $T_0$ in contact with the surface of the product initially at temperature $T_4$. A thermal flux will then pass through the sensor as described in Fig. 7.

Heat transfer modelling of the system by the quadruples method leads to the following relations $\theta(p)$ is the Laplace transform of the temperature difference $T_i(t) - T_j(t = 0)$,
for \( i = 1, 2, 3, 4; \)
\[
\begin{bmatrix}
0 \\
L[\phi_i]
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 & A_2 & B_2 & A_3 & B_3 & \theta_3 \\
C_1 & D_1 & C_2 & D_2 & C_3 & D_3 & L[\phi_i]
\end{bmatrix}
\]
\[
= \begin{bmatrix}
A & B & C & D & \theta_3 \\
\end{bmatrix}
\]
and:
\[
\begin{bmatrix}
0 \\
L[\phi_i]
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 & A_2 & B_2 & A_3 & B_3 & \theta_3 \\
C_1 & D_1 & C_2 & D_2 & C_3 & D_3 & L[\phi_i]
\end{bmatrix}
\]
where \( L[\phi_i], L[\phi_j] \) are the Laplace transforms of the heat flux transmitted at \( x = x_1 \) and \( x = x_2 \), respectively,
\[
A_i = D_i = \cosh\left(\sqrt{\frac{P}{a_i}} e_i\right); \quad B_i = \frac{1}{\lambda_i} \sinh\left(\sqrt{\frac{P}{a_i}} e_i\right);
\]
\[
C_i = \lambda_i \sqrt{\frac{P}{a_i}} S \sinh\left(\sqrt{\frac{P}{a_i}} e_i\right)
\]
\( p \) is the Laplace parameter associated to time, \( \lambda \) the thermal conductivity of the layer \( i \), \( a_i \) the thermal diffusivity of the layer \( i \), \( e_i \) the thickness of the layer \( i \) \( (e_i = x_i - x_{i-1}) \) and \( S \) the area of the thermopile cross section.

That leads to:
\[
0 = A_1 \theta_1 + B_1 L[\phi_1]
\]
\[
L[\phi_0] = C_1 \theta_1 + D_1 L[\phi_1]
\]
\[
0 = (A_1 A_2 + B_1 C_2) \theta_2 + (A_1 B_2 + B_1 D_2) L[\phi_2]
\]
\[
L[\phi_1] = (C_1 A_2 + D_1 C_2) \theta_2 + (C_1 B_2 + D_1 D_2) L[\phi_2]
\]
\[
0 = A \theta_3 + B L[\phi]
\]
\[
L[\phi] = C \theta_3 + D L[\phi]
\]
It can be deduced:
\[
L[\phi] = \frac{A}{B_1 - \frac{A_1 B_2 + B_1 D_2}{A_1 B_2 + B_1 D_2}} (\theta_3 - \theta_2)
\]

Heat transfer modelling is a measurement device such as represented in Fig. 1, which can be implemented by use of transfer functions \( f_1 \) and \( f_2 \). The initial temperature profile in the product is supposed to be linear with a slope \( \alpha \) defined as: \( \alpha = (dT_2/dx)(t = 0) \), where \( T_a \) is the surface temperature of the product. If \( \varphi(t) \) is the thermal flux flowing from the Peltier element to the product through the thermal contact resistance, it can be written:
\[
\varphi = \frac{T_3 - T_4}{R_c}
\]
and, in the Laplace space:
\[
L[\varphi] = \frac{\theta_3 - \theta_1 + \Delta T}{R_c p R_c}
\]
where \( \Delta T = T_{3(0)} - T_{4(0)} \) is the difference between the initial temperatures of the sensor and of the product surface.
The expression of the Laplace transform of the flux passing from the thermopile to the product (at a supposed uniform temperature) can be written as, according to Maillet et al. [7]:

\[ L[\varphi] = \theta_0(E_{\sqrt{\rho}} + \beta\lambda)S - \frac{\alpha\Delta S}{\rho} \]  

(10)

Combining Eqs. (7)–(10) leads finally to:

\[ L[U_p] = \frac{S}{p} \frac{\Delta T(E_{\sqrt{\rho}} + \beta\lambda) - \alpha\lambda}{f_1(p) + [Rf_1(p) - f_2(p)](E_{\sqrt{\rho}} + \beta\lambda)S} \]  

(11)

4. Sensor characterisation

The transfer functions \( f_1 \) and \( f_2 \) must be determined to characterise the sensor. Instead of trying to estimate the values of the thermal conductivities and diffusivities of the different layers of the thermopile to calculate the values of \( A_i, B_i, C_i \) and \( D_i \), the functions \( f_1 \) and \( f_2 \) have been sought directly under the classical form of a quotient of polynomials. If a flux step is applied to the sensor, the temperature \( T_i \) and the tension \( U_p \) tend towards a constant value for long time so that the transfer functions have been sought under the following form:

\[ f_1(p) = \frac{1 + a_1p + a_2p^2 + \cdots}{b_0 + b_1p + b_2p^2 + \cdots}: f_2(p) = \frac{1 + c_1p + c_2p^2 + \cdots}{d_0 + d_1p + d_2p^2 + \cdots} \]

To estimate the coefficients \( a_i, b_i, c_i \) and \( d_i \), the experimental device described in Fig. 5 has been used again. A flux step has been applied to the heating resistance and the values of \( U_p(t) \) and of \( T_i(t) \) have been recorded. If the heat flux transmitted to the insulating material is neglected, the following relations can be written:

\[ L[\varphi] = \frac{\varphi_0}{p} \frac{\Delta T(E_{\sqrt{\rho}} + \beta\lambda)}{p[f_1(p) + [Rf_1(p) - f_2(p)](E_{\sqrt{\rho}} + \beta\lambda)S]} \]

and:

\[ \theta_0 = \frac{f_2(p)}{f_1(p)} L[U_p(t)] = \frac{(1 + c_1p + c_2p^2 + \cdots)(b_0 + b_1p + b_2p^2 + \cdots)}{(d_0 + d_1p + d_2p^2 + \cdots)(1 + a_1p + a_2p^2 + \cdots)} \]

\( U_p(t) \) is then calculated by the Stehfest Laplace inversion method (as developed by Maillet et al. [7]) applied to Eq. (11) and the coefficients \( a_i, b_i, c_i \) and \( d_i \) are estimated as the values minimising the sum of the quadratic errors between the experimental and simulated values of \( U_p(t) \) and of \( T_i(t) \). This method leads to the identification of the following functions:

\[ f_1(p) = 7.722 \exp(0.2p)(1 + 18.75p + 10p^2) \]

\[ f_2(p) = 100.1(1 + 1.20p) \]

The difference between the experimental and simulated curves is lower than 0.4 K for \( T_i(t) \) and lower than 1 mV for \( U_p(t) \) that corresponds to the accuracy of the measurements and remains acceptable.

The value of \( \beta \) characterising the flux lines constriction inside the product is known for a cylindrical element with uniform heat flux or uniform surface temperature. The proposed sensor is a particular case for the following reasons:

- its section is square;
- an other constriction occurs on the internal face of the ceramic plates linked by thin thermocouples.

Under these conditions, it has been preferred to estimate the value of \( \beta \) by realising a measurement on a uniform temperature product, which thermal properties have been, determined by hot wire and hot plate measurements. The product used was cordierite (a ceramic) having thermal conductivity \( \lambda = 1.49 \text{ W m}^{-1} \text{ K}^{-1} \) (close to the conductivity of a frozen product) and thermal effusivity \( E = 1100 \text{ SI} \). The estimated value \( \beta = 108 \text{ m}^{-1} \) minimise the sum of the quadratic errors between the experimental and simulated values of \( U_p(t) \) and leads to a quite satisfactorily fitting.

5. Sensitivity analysis

The sensitivity of the tension \( U_p(t) \) to the parameters \( \Delta T \), \( \alpha \) and \( R_c \) can be deduced by derivation of Eq. (11):

\[ \frac{\dot{\alpha}(L[U_p])}{\dot{\alpha}(\Delta T)} = \frac{S}{p} \frac{\Delta T(E_{\sqrt{\rho}} + \beta\lambda)}{f_1(p) + [Rf_1(p) - f_2(p)](E_{\sqrt{\rho}} + \beta\lambda)S} \]

(12)

\[ \frac{\dot{\alpha}(L[U_p])}{\dot{\alpha}(\alpha)} = \frac{S}{p} \frac{-\lambda}{f_1(p) + [Rf_1(p) - f_2(p)](E_{\sqrt{\rho}} + \beta\lambda)S} \]

(13)
The application of the Stehfest method to these three derivatives of $U_p$ is quite sensitive to the estimation time greater than 30 s. For a variation of $\alpha$ by 100 K m$^{-1}$ from the initial value of 100 K m$^{-1}$, the corresponding variation of $U_p$ is around 2 mV. This method will only authorise the detection of slopes superior to 200 K m$^{-1}$ with a thermal contact resistance $R_c = 5 \times 10^{-4}$ m$^2$ K W$^{-1}$ taking into account noise measurement and measurement apparatus accuracy. Considering the lower value $R_c = 10^{-2}$ m$^2$ K W$^{-1}$, the limit of detection was found to be 500 K m$^{-1}$. Meanwhile, the existence of an internal temperature gradient will have a very small influence on surface temperature estimation.

This theoretical sensitivity analysis leads to two conclusions:

- the proposed sensor cannot detect low internal temperature gradients ($\alpha < 200$ K m$^{-1}$ for $R_c = 5 \times 10^{-4}$ m$^2$ K W$^{-1}$ and $\alpha < 500$ K m$^{-1}$ for $R_c = 10^{-2}$ m$^2$ K W$^{-1}$) inside a product;
- on the opposite, it may estimate with a rather good precision the surface temperature whatever the internal temperature gradient is and to detect a slope of an internal temperature profile for high values of it ($\alpha > 200$ K m$^{-1}$ for $R_c = 5 \times 10^{-4}$ m$^2$ K W$^{-1}$ and $\alpha > 500$ K m$^{-1}$ for $R_c = 10^{-2}$ m$^2$ K W$^{-1}$).

Nevertheless, if the sensor is kept in the freezing room, its temperature will be equal to the freezing temperature to be reached by the product. Then, $\Delta T$ from Eq. (11) is zero and $U_p(t)$ becomes directly proportional to the slope $\alpha$ of the internal temperature profile.
The low measured value of $U_p(t)$ may indicate that the product is totally frozen.

Furthermore, it has been verified experimentally that the reference temperature $T_0$ remains constant during the measure. Therefore, the sensitivity of $U_p(t)$ with respect to $T_0$ has not been studied.

### 6. Experimental study

The experiment described in Fig. 9 has been carried out to evaluate the applicability of the proposed method.

The electrical power produced in the heating resistance is equal to was measured to evaluate the value of the thermal flux passing through the tested material since their are equal if the thermal losses through the insulating material is neglected. The heating resistance was kept on heating until a quasi-stationary regime was reached (temperature varying less than 0.2 K during 5 min) to ensure the existence of a linear internal temperature profile which slope is thus given by:

$$\alpha = $ \frac{\varphi}{lS}$$

where $\varphi$ is the heat flux produced by the heating element, $\lambda$...
the thermal conductivity of the tested product and \( S \) the product area (equal to heating resistance area).

The copper piece was then taken off and immediately replaced by the sensor whose tension \( U_p(t) \) was recorded during 60 s. Fig. 10 represents several recordings of \( U_p(t) \) obtained with four different fluxes corresponding to four different internal temperature profile slopes. It can be noted that these curves are almost straight between 30 and 60 s and that the slope of the curves \( U_p(t) \) between 30 and 60 s are correlated to the internal temperature profile slope as shown in Fig. 11.

7. Conclusion

The modelling of heat transfer in the sensor during a measurement and the determination of the transfer functions between temperatures and fluxes lead to satisfactorily simulation of the transient tension \( U_p(t) \) delivered by the sensor as shown by tests realised with cordierite. A sensitivity analysis has set the application limits of the proposed sensor. The influence of the slope of a linear internal temperature profile on the transient tension \( U_p(t) \) has been experimentally evidenced.

Meanwhile, the estimated transfer functions do not lead to an exact simulation of the experimental results for \( U_p(t) \) since the residues are not randomly dispersed around a zero value, other forms of transfer functions leading to a best fitting are so to be found. The inverse problem resolution method for the estimation of the surface temperature and the slope of the internal temperature profile is also to be precised and tested.

Nevertheless, a simple practical application could yet be thought by keeping the sensor inside the freezing room, a null of very weak variation the thermopile tension \( U_p(t) \) when set in contact with the product would mean that the product is uniformly at the freezer temperature.

References