Shrinkage and density evolution during drying of tropical fruits: application to banana

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Abstract

This article presents results on the variation in density and shrinkage of banana during its drying. Volume and density measurements were carried out on three series of samples with average initial moisture contents of 506, 450 and 393 %kg(kg db)$^{-1}$ and dried at 40, 50 and 60 °C respectively. Each series contained 16 samples and drying was interrupted at different moments to obtain various moisture contents. By considering that the variation in the product volume was equal to the volume of the evaporated water, mathematical models were proposed. Calculated and experimental values were in good agreement, with a maximum standard error (at 40 °C) of 19 kg m^{-3} for density and of 3.7×10^{-8} m^3 for volume. The results may improve the modelling of the drying kinetics of this product and the determination of its various characteristics.
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1. Introduction

As for most foodstuffs, dehydration of fruits and vegetables (banana in this particular case) produces large changes in their volume and their heat and mass exchange area because of spatial and time variable structure. It was found that, during such an operation, the decrease of their surface under the influence of the contraction of tissues compensates partially for the loss of water. Consequently, all the parameters which depend on internal and external dimensions change, making the use of a complete model for drying simulation of these products difficult. The very low number of reliable data on this phenomenon makes the concept of the characteristic curve of drying one of the best adapted method used by many authors (Belahmidi, Belghit, Mrani, & Kaoua, 1993; Desmorieux, 1992; Fornell, 1979; Keey, 1977; Talla, Jannot, Kapseu, & Nganhou, 2001) to describe the behavior of the foodstuffs during drying.

The shrinkage phenomenon affects in particular the diffusion coefficient of the material, which is one of the main parameters governing the drying process; it also has an influence on the drying rate (Lima, Queiroz, & Nebra, 2002; Queiroz, 1994; Queiroz & Nebra, 1996). Besides, various characteristics of the material depends on its density so that the knowledge of the density variation with moisture content will be useful to characterize the behavior of this material. The determination of its variation is thus essential (Nadeau & Puiggali, 1995).

Several theoretical and experimental studies were carried out to analyze and foresee the mass transfer in foodstuffs; the drying of banana is a good illustration (Bowrey, Buckle, Hamey, & Pavenayotin, 1980; Drouzas & Schubert, 1996; Kiranouidis, Tsami, Maroulis, & Marinos-Kouris, 1997; Krokida, Maroulis, & Marinos-Kouris, 1998; Mauro & Menegalli, 1995; Prasertsan & Saen-sabv, 1998; Rastogi, Raghavarao, & Niranjan, 1997; Schirmer, Janjai, Esper, Smitabhindu, & Mûhhlbauer, 1996; Talla et al., 2001). However, few works were carried out on the drying kinetics of banana taking into account the shrinkage phenomenon (Coutinho, Alsina, & Silva, 1997; Krokida & Maroulis, 1997; Lima et al., 2002; Queiroz, 1994; Queiroz & Nebra, 1996). These works however pointed out that the shrinkage phenomenon has a strong influence on the drying rate of banana by modifying significantly its diffusion coefficient.

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However, these authors do not worry about the influence of the initial moisture content or about temperature on the shrinkage phenomenon. Studies related to the density variation of banana during drying could not be found in the literature.

The first objective in this study is the development of mathematical models for density and shrinkage variation of banana during drying. The second objective is the validation of these models on the basis of an experimental study on this fruit. The influence of the initial moisture content and the temperature is also analyzed.

2. Theoretical analysis

To elaborate on the mathematical models, the following hypotheses were considered:

- The variation of the volume corresponding to the shrinkage of the product is equal to the evaporated volume of water.
- The product is constituted of a solid structure with density ρ_d and volume V_d, pores of which are occupied by water with density ρ_w and volume V_w.

From these two hypotheses and by considering m as the mass of the product at a given time, m_w the mass of water contained in this product and m_d the mass of its solid materials, it can be said that:

$$m = m_w + m_d; \quad V = V_w + V_d \tag{1}$$

Furthermore, it can be written as:

$$\rho = \frac{m}{V} \quad \text{and} \quad X = \frac{m_w}{m_d} \tag{2}$$

X: moisture content of the product in dry basis $(\% \text{kg} (\text{kg db})^{-1})$.

The development of Eqs. (1) and (2) gives the following expressions:

$$\rho = \rho_d \frac{\rho_w(1+X)}{\rho_w + \rho_d X} \quad \text{and} \quad V = V_0(A + \beta X) \tag{3}$$

with

$$A = \frac{1}{1 + aX_0}; \quad \beta = \frac{a}{1 + aX_0} \quad \text{and} \quad a = \frac{\rho_d}{\rho_w} \tag{4}$$

X_0 represents the initial moisture content, V_0 the initial volume and β a parameter called shrinkage coefficient.

If $(X_0 - X)$ is chosen to quantify at time t the quantity of water evacuated from the product by unity of dry mass, the expressions (3) become in reduced forms:

$$V_r = 1 - \beta(X_0 - X) \quad \text{and} \quad \rho_r = \frac{1 - \frac{1}{1+X_0}(X_0 - X)}{1 - \beta(X_0 - X)} \tag{5}$$

with

$$V_r = \frac{V}{V_0} \quad \text{and} \quad \rho_r = \frac{\rho}{\rho_0} \tag{6}$$

Eq. (5) verify the following conditions in the limits:

For $X = X_0$: $\quad \rho = \rho_0 = \rho_d \dfrac{\rho_w(1 + X_0)}{\rho_w + \rho_d X_0} \quad$ and

$$V = V_0 = V_d \left(1 + \frac{\rho_d}{\rho_w} X_0\right) \tag{7}$$

For $X = 0$: $\quad \rho = \rho_d \quad$ and $\quad V = V_d \tag{8}$

3. Experimental

3.1. Equipment

The curves representing the density and shrinkage of banana during its drying were experimentally

determined under constant conditions of temperature. The experimental equipment included:

- The die which was used for making samples of identical dimensions. The diameter of each sample was constant and equal to 25 mm. The thickness was 6 mm.
- Small dishes which were used as supports for samples during the drying process.
- An electronic balance with precision of 10^{-3} g.
- A dehydration oven whose temperature was fixed at 105 °C, to yield the dry mass of a sample after 48 h.
- A drying oven regulated at temperatures of 40, 50 and 60 °C respectively for this study, allowing controlled decrease of moisture content of the tested samples.
- A volume gauge with mercury, the system was based on the properties of the mercury (flow and high density), so as to measure the volume of the tested sample.

3.2. Method of sample treatment

Small dishes, numbered beforehand in the form of couple (iA, iB), iA being the sample number i placed in the regulated drying oven (control of the moisture content) and iB reserved for the dehydration oven (extraction of dry mass), were weighed. The bananas were manually washed, peeled and cut, at first, in slices of thickness more than double the chosen test thickness (6 mm in our case). Each piece was then split into two (always in the form of slice). By means of die, these two new fragments supplied two cylindrical samples of identical dimensions that were set respectively in small dishes iA and iB. The procedure was repeated by making i to vary from 1 to n (number of experimental points, 16 in the case of each testing).

The small dishes and their contents were again weighed and then introduced into oven (series A in the regulated drying oven and series B in the dehydration oven). After 48 h, samples placed in the dehydration oven at 105 °C were weighed and the initial moisture contents determined. On the other hand in the drying oven where temperature for the three tests was 40, 50 and 60 °C respectively, samples were taken out intermittently to obtain a set of samples covering all the ranges of the moisture contents (from X_0 in some % kg (kg db)$^{-1}$). These values of temperature are recommended for drying tropical fruits preserving the best ratio of organoleptic component of the product obtained and duration of drying (Talla et al., 2001). Each sample was then weighed before its volume was measured in the volume gauge with mercury.

For that purpose, the piston was moved to coincide with the level of mercury with a fixed index and the "zero" value (in the absence of product) indicated by the dial was set. The sample was then introduced into the cylinder by blocking it in a stable position by an index card and the same manipulation carried out before setting the new value indicated by the dial. The difference between the new value and the value with "zero" (without sample) gives the volume of the sample knowing the cross section of the piston.

By repeating this procedure for 16 experimental points, one can deduce the experimental variation of the density and/or the shrinkage of the product with its moisture content.

3.3. Estimation of coefficient of shrinkage β

An estimation β_i of the shrinkage coefficient β was calculated for each $(n-1)$ couple of experimental points (V_{ri}, X_i) by using the following relation deduced from Eq. (5):

$$\beta_i = \frac{1 - V_{ri}}{X_{0i} - X_i} \tag{9}$$

The first couple of experimental values (V_{r0}, X_0) were not used because it does not allow the application of formula (9).

The method of linear least squares, usually used by the experimenters, would lead to estimate the coefficient β by the expression:

$$\beta = \frac{1}{n-1} \sum_{i=1}^{n-1} \beta_i \tag{10}$$

However, this method gives to all the values β_i the same weight independently of the uncertainty with which they are known. Measurement results known with high uncertainty can then cause errors in the final result. When the estimation errors of such a parameter are not constant, as in this case, the application of the Gauss–Markov method reduces the influence of the highly noisy experimental points (Jannot, Batsale, Ahouannou, Kanmogne, & Talla, 2002). If furthermore, as in our case, the measures are not correlated (no interference between the various measures), the simplified Gauss–Markov method can be applied which leads to the following expression for the estimation of β:

$$\beta = \frac{\sum_{i=1}^{n-1} \frac{\beta_i}{(\Delta\beta_i)^2}}{\sum_{i=1}^{n-1} \frac{1}{(\Delta\beta_i)^2}} \tag{11}$$

where $\Delta\beta_i$ is the estimation uncertainty of β_i.

The standard deviation on the estimated value of β is given, according to Beck (1977), by the relation:

$$\Delta\beta = \sqrt{\frac{1}{\sum_{i=1}^{n-1} \frac{1}{(\Delta\beta_i)^2}}} \tag{12}$$

The calculation of the uncertainties of estimation of β_i is detailed in the following paragraph.

3.4. Calculation of uncertainties

3.4.1. Measurement of uncertainty on the moisture content

The uncertainty on the moisture content is due to:

- The uncertainties on the mass measurements of samples which were used for the evaluation of the initial moisture content (dry mass);
- The uncertainty on the volume measurement for a given moisture content.

We have

$$X = \frac{m}{m_0}(1 + X_0) - 1 \quad \text{and} \quad X_0 = \frac{m'_0 - m'_d}{m'_d} \tag{13}$$

where from

$$X = \frac{m}{m_0}\left[1 - \frac{m'_0 - m'_d}{m'_d}\right] - 1 \tag{14}$$

Differentiation of relation (14), leads to the expression:

$$\Delta X = (1 + X)\left[\frac{1}{m} + \frac{1}{m_0} + \frac{1}{m'_0} + \frac{1}{m'_d}\right]\Delta m \tag{15}$$

3.4.2. Measurement of uncertainty on the volume

The measurement of uncertainty of the volume results essentially from the uncertainty on the value of the initial volume for the various samples. If the sampling is carried out by means of a die to guarantee the same dimension, the homogeneity of the product is however not sure. It can be written as:

$$\Delta\left[\frac{V}{V_0}\right] \bigg/ \left[\frac{V}{V_0}\right] = \frac{\Delta V}{V} + \frac{\Delta V_0}{V_0} \approx \frac{\Delta V_0}{V_0} \tag{16}$$

where from

$$\Delta\left[\frac{V}{V_0}\right] \approx \frac{V}{V_0}\frac{\Delta V_0}{V_0} \tag{17}$$

3.4.3. Measurement of uncertainty on the density

The estimation of uncertainty on the density mainly results from the mass measurement of uncertainty, the volume measurement of uncertainty in the volume gauge being negligible. So that:

$$\rho = \frac{m - M}{V} \tag{18}$$

m being the raw mass and M the tare.

Differentiation of expression (18) leads to:

$$\Delta\rho = \rho\left(\frac{2\Delta m}{m - M} + \frac{\Delta V}{V}\right) \approx \rho\frac{2\Delta m}{m - M} \tag{19}$$

3.4.4. Measurement of uncertainty on the shrinkage coefficient

The estimation of uncertainty on the shrinkage coefficient results from measurement of uncertainties on the initial volume and on the moisture content directly related to the mass. Differentiation of Eq. (9) leads to:

$$\Delta\beta_i = \beta_i\left[\left(\frac{V}{V_0}\right)_i\frac{\Delta V_{0i}}{V_{0i} - V_i} + \frac{\Delta X_{0i} + \Delta X_i}{X_{0i} - X_i}\right] \tag{20}$$

3.5. Treatment of experimental results

The application of the experimental method described above yielded the experimental points corresponding to the sample volumes for various moisture contents. These volumes combined with the sample masses allowed the calculation of the corresponding densities. The curves in Figs. 1 and 2 represent, respectively the variation of the density and of the volume of banana according to the moisture content. The main initial and final mean values of the three tests are recorded in Table 1.

Table 2 presents the theoretical and estimated values of the shrinkage coefficient (using respectively Eqs. (4), (9), (11) and (20)) for the various experimental conditions as well as the estimated uncertainties on this parameter.
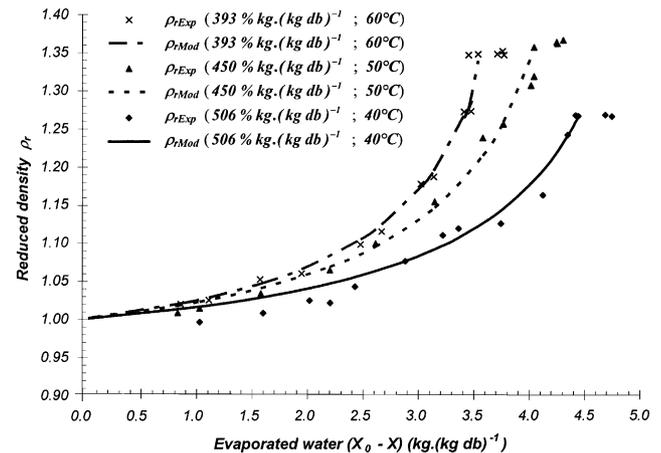

Fig. 1. Comparison between the experimental and predicted densities of banana obtained during drying.
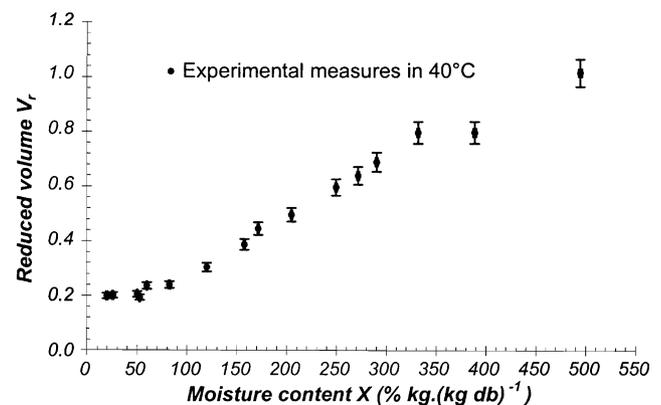

Fig. 2. Bars of error on measure of volume at 40 °C.

Table 1
Experimental conditions

Testing	θ (°C)	Initial values			Final values		
		$\overline{X}_0$ (% kg (kg db)$^{-1}$)	$10^6 V_0$ (m^{-3})	ρ_0 (kg m^{-3})	$\overline{X}_f$ (% kg (kg db)$^{-1}$)	$10^6 V_f$ (m^3)	ρ_f (kg m^{-3})
1	40	506	2.645	1093	21	0.525	1 383
2	50	450	2.940	1076	15	0.450	1 395
3	60	393	2.850	1028	14	0.495	1 384

Table 2
Values of coefficient of shrinkage

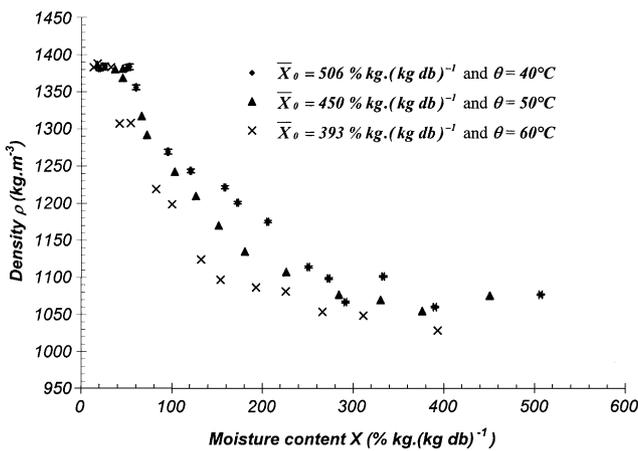| Testing	θ (°C)	$\overline{X}_0$ (% kg (kg db)$^{-1}$)	β_{theo}	β	$\Delta\beta$	$|\beta - \beta_{\text{theo}}|$	$\frac{|\beta - \beta_{\text{theo}}|}{\beta}$
1	40	506	0.1732	0.1709	0.0013	0.0023	1.3%
2	50	450	0.1918	0.1974	0.0012	0.0056	2.8%
3	60	393	0.2153	0.2231	0.0013	0.0078	3.5%

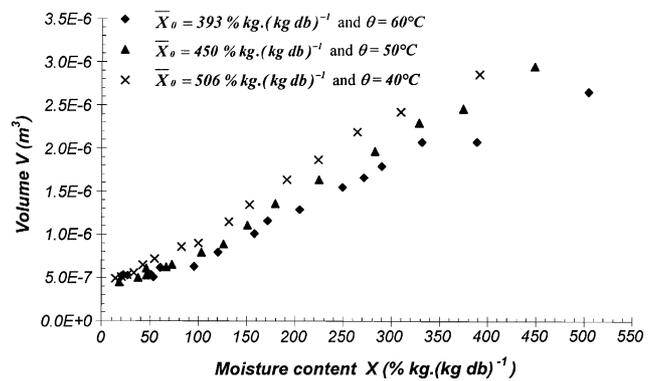Fig. 3. Experimental values of density of banana according to the moisture content.

Fig. 4. Experimental values of volume of banana according to the moisture content.

The measurement of uncertainty of the masses is supplied by the builder of the balance and its value is 10^{-3} g. As for the measurement of the volume V_0, a die was used to make 10 samples of banana, the volumes of which being measured in the volume gauge with mercury in their initial state. From these measurement results, a maximal distance (that we shall assimilate to the uncertainty of measure) of 0.15 cm^3 were found around the mean value.

From the relations previously developed, the error on each measurement point can be calculated. That allows to set the error bars on the various measurement points as done for example in Fig. 3 for the density and in Fig. 2 for the volume, testing being realized at 40 °C.

4. Results and discussion

4.1. Banana volume

Fig. 4 represents the experimental points of the volume of banana according to its moisture content. It can

be seen that the volume of this product evolves in the same sense as the moisture content. This result is also predictable, just as banana shrinkage during drying. This figure also highlights the influence of the moisture content on the shrinkage of banana. The obtained values (of $\beta = 0.171$ for $\overline{X}_0 = 506$ % kg (kg db)$^{-1}$ to $\beta = 0.223$ for $\overline{X}_0 = 393$ % kg (kg db)$^{-1}$) are in agreement with the works of Lima et al. (2002) who obtained a shrinkage coefficient equal to 0.269 for a drying experiment in 39.9 °C carried out on banana with initial moisture content 316 % kg (kg db)$^{-1}$; the above elaborated theoretical model foresees in these conditions a value of 0.258. For weak moisture contents, one can also note the convergence of all the experimental curves towards the same volume which represents the volume of the solid material. Fig. 5 compares the experimental values and the values predicted by the model. This model allows to reproduce in a very satisfactory way the measured values: the maximum standard error (in 40 °C) between the theoretical and experimental curves representing the volume is 3.7×10^{-2} cm^3.
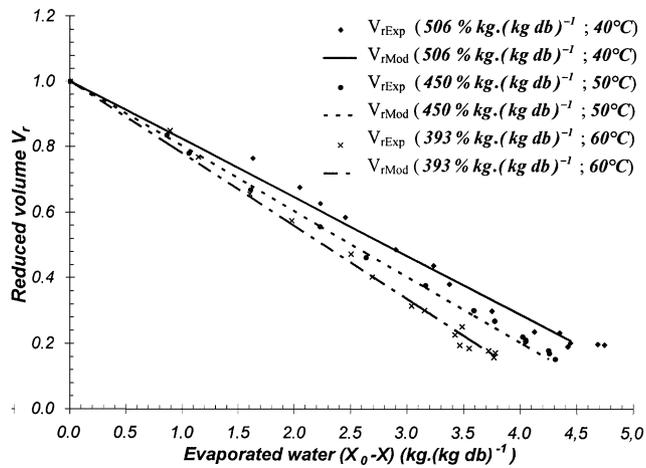
Fig. 5. Comparison between the experimental and predicted volumes of banana obtained during drying.

4.2. Banana density

The experimental results represented in Fig. 3 shows that for important moisture contents, the densities are close to water density which varies between 997 and 983 $kg\,m^{-3}$ when the temperature varies from 20 to 60 °C. This result is predictable as far as the fresh banana possesses a very high moisture content (on average 450 $\%\,kg\,(kg\,db)^{-1}$); the stiff material mass is rather negligible compared with the water mass and consequently the density of the sample is very close to the density of the liquid in the product.

As the moisture content decreases in the product, the density increases. So that it can be concluded that in the case of banana which is a shrinkable material, the density of the constituent of solid matrix is superior to that of the extracted solvent because the solid replaces the liquid. This result is in perfect agreement with the literature (Nadeau & Puiggali, 1995).

The influence of initial moisture content on the evolution of the density of banana during drying can also be noticed in the same figure: all the experimental curves converge to the same point (situated at approximately $X = 0\ \%\,kg\ water\,(kg\,dm)^{-1}$ and $\rho \approx 1400\ kg\,m^{-3}$) for weak moisture contents. The density at this point represents the density of the solid matrix. The simulation by the above elaborated model leads to good agreement with experimental results, as shown in Fig. 1: the maximum standard error (at 40 °C) between theoretical and experimental curves representing density is 19 $kg\,m^{-3}$.

Table 2 shows that the estimation of uncertainty $\Delta\beta$ on the shrinkage coefficient remains lower than $(\beta - \beta_{\text{theo}})$. The increase of this deviation with the temperature (Table 2) could be explained by the existence of the thermal effect on the shrinkage of this product during the drying. With a maximum relative deviation lower than 4% (attempt in 60 °C), it can however be concluded that this effect remains weak.

5. Conclusion

The fundamental equations of shrinkage and the evolution of the density of banana during the drying are developed on the basis of the hypothesis that the variation of the volume of the product corresponds to the volume of evaporated water. Analyzing the obtained results leads to the following conclusions:

- The proposed theoretical model for banana shrinkage supplies data in good agreement with the experiment. This result may improve the modelling of the drying kinetics of this fruit by taking into account the changes of exchange area according to the moisture content.
- The experimental and theoretical data show the dominating influence of the initial moisture content with regard to the temperature on the shrinkage coefficient of this product. The influence of the temperature on the shrinkage phenomenon can thus be neglected in the simulation models of the drying kinetics with an acceptable error (lower than 4%).
- The rather good prediction of banana density depending on its moisture content by the proposed model may improve the determination of the various characteristics of this fruit depending on its density.

These models may be useful in the description of the drying process of banana characterized by variable properties during time and conditions in the limits which modify, making the use of an analytical solution quite difficult and leading to the use of comportment model.

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