

# Simplified estimation method for the determination of the thermal effusivity and thermal conductivity using a low cost hot strip

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## Abstract

This paper presents the study of a hot strip made of thin rectangular electrical resistance with a thermocouple placed on its centre. The purpose was to simultaneously estimate thermal effusivity and conductivity in a limited time ( $t_2 < 180$  s) using a low cost probe. Heat transfer has been modelled with the quadrupole formalism to simulate the evolution of the temperature at the centre of a hot strip set between two samples of material to be characterized when a heat flux step is applied. Simulation is used to fix the optimal dimensions of a hot strip that behaves as a hot plate (1D transfer) during a minimal time  $t_1$  ( $> 20$  s) and that has higher sensitivity to the thermal conductivity between  $t_1$  and  $t_2$  (2D transfer). The thermal effusivity is estimated between 0 and  $t_1$  by minimization of the quadratic errors between the experimental curve and the curve calculated by the classical hot plate model. The thermal conductivity is estimated between  $t_1$  and  $t_2$  but using the complete 2D model. To validate the model and the estimation process, experimental tests were realized on three materials with low diffusivities ( $a < 2 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ ) and having typical area of  $6 \text{ cm} \times 4 \text{ cm}$  and typical thickness of 1.5 cm.

**Keywords:** hot strip, transient method, thermal quadrupoles, effusivity, conductivity, parameter estimation method

## Nomenclature

$a$	thermal diffusivity ( $\text{m}^{-2} \text{ s}^{-1}$ )	$\ell$	half-length of the hot-strip (m)
$b$	half-width of the hot strip (m)	$L$	sample half-width (m)
$c_s$	thermal capacity of the hot strip ( $\text{J K}^{-1} \text{ }^\circ\text{C}^{-1}$ )	$L_{\text{hw}}$	hot wire length (m)
$e$	sample thickness (m)	$m_s$	half-mass of the hot strip (kg)
$E$	thermal effusivity ( $\text{J m}^{-2} \text{ }^\circ\text{C}^{-1} \text{ s}^{-1/2}$ )	$n_F$	number of terms in the sum for Fourier transform inversion
$K_0$	modified Bessel function of the second kind of order zero	$n_L$	number of terms in the Stehfest algorithm for Laplace transform inversion
$K_1$	modified Bessel function of the second kind of order one	$p$	Laplace parameter
		$r_{\text{hw}}$	hot wire radius (m)
		$R$	electrical resistance ( $\Omega$ )

$R_c$	thermal contact resistance ( $^{\circ}\text{C W}^{-1}$ )
$T_s$	temperature increase of the hot strip ( $^{\circ}\text{C}$ )
$T_{hw}$	temperature increase of the hot wire ( $^{\circ}\text{C}$ )
$V_i$	coefficients of the Sthefest algorithm
$\theta$	Laplace transform of the sample temperature
$\theta_c$	finite cosine Fourier transform of $\theta$
$\theta_{cs}$	finite cosine Fourier transform of $\theta_s$
$\theta_{hw}$	Laplace transform of the temperature at the centre of the hot wire
$\theta_s$	Laplace transform of the temperature at the centre of the hot strip
$\phi$	heat flux dissipated in the hot strip (W)
$\Phi_{cs}$	Laplace transform of the heat flux dissipated in the hot strip
$\lambda$	thermal conductivity ( $\text{W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$ )

## 1. Introduction

The aim of this study was to develop a relatively fast method (time duration lower than 180 s) using a low cost probe to estimate the thermal effusivity and the thermal conductivity simultaneously from a single temperature recording. Several types of probes designed to estimate simultaneously thermal effusivity and conductivity have already been studied. The estimation is always based on the analysis of transient temperature measurement of a heating probe set between two samples of the material to be characterized. The main probes among these are:

- The hot strip (HS) that is a heating strip made of an electrical resistance disposed on a thin rectangular support; it has been studied by Gustafsson *et al* [1], Lourenco *et al* [2] and Song *et al* [3]. The electrical resistance is used both to produce heat and to measure the mean temperature of the hot strip. The modelization of the heat transfer enables the estimation of both thermal effusivity and thermal conductivity by processing the transient measurement of voltage and current within the resistance. The major disadvantage of this method is the relatively high cost of the equipment. This method also requires estimating a time delay representing the heat capacity effect of the strip [4].
- The hot disc (HD) developed by Gustafsson [4] is analogous to the HS but the probe has a circular shape. Its disadvantages are the same as those of the hot strip.
- The parallel probes as described by Hladik [5] where temperature is measured at a distance  $r$  (approximately 1 cm) of a linear hot wire. The distance  $r$  must be known with a high precision for a precise estimation of the thermal diffusivity.
- The heating ring studied by Cull [6] where temperature is measured at the centre of a circle (approximately 1 cm diameter) was made of a thin heating wire. This probe is not easy to realize and the diameter of the ring must be known with a high precision for a precise estimation of the thermal diffusivity. The estimation process requires determining the maximum of the derivative of the probe temperature that could lead to some uncertainty.
- The hot strip with an imposed surface temperature on the unheated face of the samples as studied by Ladevie

[7]. According to Ladevie [7], its sensitivities to thermal conductivity and thermal diffusivity are higher than those of the dynamic plane source (DPS) or extended dynamic plane source (EDPS) studied by Malinarič [8]. Its main disadvantages are its duration (typically 10 min), a rather complex data processing method and the necessary evaluation of convective heat losses on the lateral surfaces of the samples.

Methods 1–4 used the hypothesis for samples of a semi-infinite medium so that the estimation of the convective losses on the lateral surfaces is not necessary. In methods 3 and 4, the distance between the heating wire and the temperature sensor must be known with a high precision that leads to uncertainty in the estimated values and a more complex realization. Method 5 uses a local temperature measurement at the centre of the strip but has longer time duration and the associated data process is more complex.

The proposed method uses a rectangular-shaped electrical resistance on which a thermocouple with thin wires is stuck. Temperature is measured at the centre of the resistance making it unnecessary to take into account thermal losses through electrical wires at one end of the resistance. The resistance is inserted between plane surfaces of two samples of the material to be characterized. The sample dimensions are large enough to verify the hypothesis of the semi-infinite medium in all directions during the experiment. The ratio length/width for the resistance is chosen so that heat transfer at its centre may be considered as bidirectional during a minimal time of 180 s. The method consists in using the beginning of the temperature recording during a time interval  $t_1$  when heat transfer at the centre of the resistance is unidirectional to estimate the thermal effusivity of the samples with a hot plane model. A complete modelization of bidirectional transfer coupled with a parameter estimation method is then used to estimate thermal conductivity from the whole temperature recording.

The advantages of this method are the very low cost of the probe and a rather simple data processing method based on separated heat transfer models at the centre of the hot strip:

- unidirectional (1D) during a time  $t_1$  for thermal effusivity estimation (hot plane model),
- bidirectional (2D) between times  $t_1$  and  $t_2$  for conductivity estimation.

## 2. Modelization

### 2.1. Complete model

The experimental device is represented in figure 1. Temperature increase  $T_s(x, y, t)$  at coordinate point  $(x, y)$  of the hot strip verifies the following relation during time  $t_2$  when heat transfer is still bidirectional (infinite hot strip):

$$\frac{\partial^2 T_s(x, y, t)}{\partial x^2} + \frac{\partial^2 T_s(x, y, t)}{\partial y^2} = \frac{1}{a} \frac{\partial T_s(x, y, t)}{\partial t} \quad (1)$$

with the following boundary conditions:

- (1) at  $y = 0$ :  $-\lambda \frac{\partial T_s(x, y, t)}{\partial y} = -\phi$  if  $x < b$  and  $-\lambda \frac{\partial T_s(x, y, t)}{\partial y} = 0$  if  $x > b$ ,
- (2) at  $x = 0$ :  $\frac{\partial T_s(0, y, t)}{\partial x} = 0$  for symmetry reasons,
- (3) at  $x = L$ :  $T_s(L, y, t) = 0$  hypothesis of semi-infinite medium in the Ox direction,

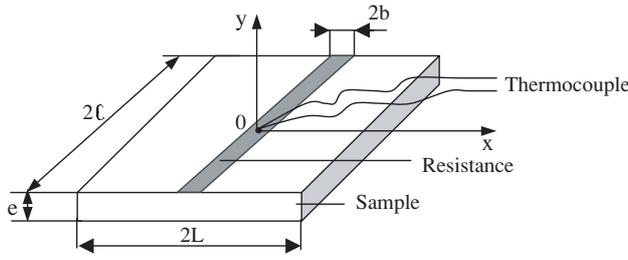


Figure 1. Experimental device.

(4) at  $y = e$ :  $T_s(x, e, t) = 0$  hypothesis of semi-infinite medium in the Oy direction,

where  $b$  is the half-width of the hot strip (m),  $e$  is the thickness of the sample (m) and  $L$  is the half-width of the sample (m).

Upon application of a Laplace transform followed by a cosine Fourier transform between  $x = 0$  and  $x = L$ , relation (1) becomes

$$\frac{\partial^2 \theta(x, y, p)}{\partial x^2} + \frac{\partial^2 \theta(x, y, p)}{\partial y^2} = \frac{p}{a} \theta(x, y, p) \quad (2)$$

then

$$\frac{d^2 \theta_c(n, y, p)}{dy^2} = \left( \frac{p}{a} + \frac{n^2 \pi^2}{L^2} \right) \theta_c(n, y, p). \quad (3)$$

The general solution of equation (3) can be written as

$$\theta_c(n, y, p) = C_1 \cosh(q_n y) + C_2 \sinh(q_n y)$$

with

$$q_n^2 = \frac{p}{a} + \frac{n^2 \pi^2}{L^2}.$$

By using the quadrupole formalism as presented by Ladevie [7] and considering the hot strip as a thin system (no thermal gradient in the Oy direction), the Laplace transform  $\theta_{cs}$  of the temperature increase and  $\Phi_{cs}$  of the half-flux at the centre of the hot strip can be written as a matrix equation:

$$\begin{bmatrix} \theta_{cs}(n, 0, p) \\ \Phi_{cs}(n, 0, p) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ m_s c_s p & 1 \end{bmatrix} \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_c(n, 0, p) \\ \lambda q_n S \theta_c(n, 0, p) \end{bmatrix}$$

where  $m_s$  is the half-mass of the strip (kg),  $c_s$  is the heat capacity of the strip ( $\text{J K}^{-1} \text{kg}^{-1}$ ),  $S$  is the strip area ( $\text{m}^2$ ) and  $R_c$  is the thermal contact resistance ( $\text{W}^\circ\text{C}^{-1}$ ).

Or

$$\theta_{cs}(n, 0, p) = \frac{1 + \lambda R_c S q_n}{m_s c_s p (1 + \lambda R_c S q_n) + \lambda S q_n} \frac{\sin(\alpha_n b)}{\alpha_n} \frac{\phi}{p} \quad (4)$$

with

$$q_n = \sqrt{\frac{p}{a} + \frac{n^2 \pi^2}{L^2}} \quad \text{and} \quad \alpha_n = \frac{n\pi}{L}.$$

An inverse Fourier transform leads to

$$\theta_s(0, 0, p) = \frac{1}{L} \theta_{cs}(0, 0, p) \frac{2}{L} \sum_{n=1}^{\infty} \theta_{cs}(n, 0, p). \quad (5)$$

Practically, this infinite sum is calculated for a finite number  $n_F$  of terms, typically between 100 and 1000.

Applying the Steffest algorithm to calculate the inverse Laplace transform of the temperature  $\theta_s$  finally leads to

$$T_s(0, 0, t) = \frac{\ln(2)}{t} \sum_{j=1}^{n_L} V_j \theta_s \left( 0, 0, \frac{j \ln(2)}{t} \right). \quad (6)$$

## 2.2. Simplified model

When neglecting the thermal contact resistance and the hot strip mass, the temperature increase at the centre of the hot strip has the following expression according to Carslaw [9]:

$$T_s(0, 0, t) = \frac{2\phi\sqrt{at}}{\lambda\sqrt{\pi}} \left[ \operatorname{erf} \left( \frac{b}{2\sqrt{at}} \right) - \frac{b}{2\sqrt{\pi at}} \operatorname{Ei} \left( -\frac{b^2}{4at} \right) \right] \quad (7)$$

with

$$\operatorname{Ei}(-x) = \int_x^{\infty} \frac{e^{-t}}{t} dt.$$

The expression  $\frac{2\phi\sqrt{at}}{\lambda\sqrt{\pi}}$  represents the temperature of a hot plane if the thermal contact resistance and its mass were neglected.

## 3. Sensitivity study

The complete model has been used to calculate the sensitivity of the temperature increase  $T_s(t)$  at the centre of a hot strip, using the parameters  $E$ ,  $\lambda$ ,  $R_c$  and  $m_s c_s$ . The aim of this sensitivity study realized for three materials is

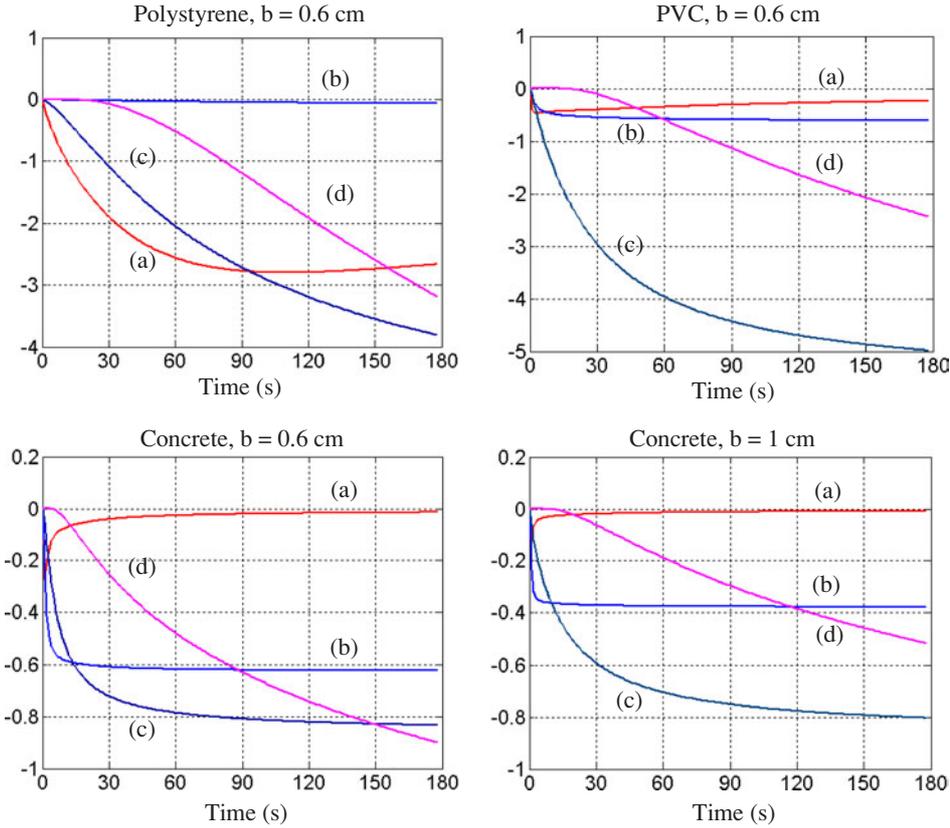
- to estimate the time  $t_1$  during which the hot strip behaves as a hot plane;
- to verify that between 0 and  $t_1$  the parameters  $E$ ,  $R_c$  and  $m_s c_s$  have no correlated influence on  $T_s$  so that they could be estimated properly on this interval;
- to verify that the sensitivity of  $T_s$  to  $\lambda$  is sufficiently high to enable a precise estimation of  $\lambda$ .

Calculations have been made for a 1.2 cm wide hot strip on the following materials: polystyrene ( $\lambda = 0.037 \text{ W m}^{-1} \text{ K}^{-1}$ ,  $E = 42.4 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}$ ), PVC ( $\lambda = 0.21 \text{ W m}^{-1} \text{ K}^{-1}$ ,  $E = 530 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}$ ) and concrete ( $\lambda = 1.4 \text{ W m}^{-1} \text{ K}^{-1}$ ,  $E = 1683 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}$ ). The following values were considered in this sensitivity study:  $m_s c_s = 0.2 \text{ J}^\circ\text{C}^{-1}$ ,  $S R_c = 0.002 \text{ m}^2 \text{ }^\circ\text{C W}^{-1}$ . Results are reported in figure 2.

For polystyrene, hot plane behaviour was noted up to  $t_1 = 30 \text{ s}$  approximately. However, the sensitivity to  $m_s c_s$  is appreciable during this period and almost proportional to the sensitivity to  $E$ . It will thus be difficult to estimate  $E$  with precision between 0 and  $t_1$  in this case. Consequently, it would be necessary to use a broader hot strip to exploit a zone where the influences of the parameters  $m_s c_s$  and  $E$  are less correlated, with time measurement being greater than 180 s. Sensitivity to the parameter  $\lambda$  is sufficiently high for a correct estimation after determination of the other parameters.

For PVC, hot plane behaviour could be noted up to  $t_1 = 25 \text{ s}$  approximately with an appreciable sensitivity to  $E$ . This sensitivity to  $E$  is not correlated with the sensitivities to the other parameters. Sensitivity to  $\lambda$  is also sufficient for a correct estimation after determination of the other parameters.

For concrete, the hot plane behaviour lasts only 6 s with a hot strip having a half-width  $b = 0.6 \text{ cm}$ . During this time interval, the sensitivity to the parameters  $E$ ,  $R_c$  and  $m_s c_s$  is strongly correlated which makes their estimation quite difficult and inaccurate. But, with a half-width  $b = 1 \text{ cm}$ , the hot plane behaviour lasts approximately 15 s with a decorrelation of the parameters thus making the estimation of  $E$  possible between



**Figure 2.** Sensitivity (in K) of  $T_s(0, 0, t)$  to the parameters  $E$ ,  $\lambda$ ,  $R_c$  and  $m_s c_s$  for different materials: (a)  $m_s c_s \frac{\partial T_s}{\partial (m_s c_s)}$ , (b)  $R_c \frac{\partial T_s}{\partial R_c}$ , (c)  $E \frac{\partial T_s}{\partial E}$  and (d)  $\lambda \frac{\partial T_s}{\partial \lambda}$ .

0 and  $t_1$ . Sensitivity to the parameter  $\lambda$  is sufficiently high for a correct estimation between  $t_1$  and 180 s.

As a conclusion, the proposed method can be applied to medium conductivity materials such as PVC or concrete with a time measurement of 180 s. Low conductivity materials such as polystyrene may also be characterized with this method but a hot strip with a width of several centimetres will be necessary and the time measurement cannot remain as short as 180 s.

## 4. Parameter estimation method

### 4.1. Thermal effusivity estimation

During time  $t_1$  when the heat transfer at the centre of the hot strip remains unidirectional, the temperature at the centre of the hot strip evolves like that of a hot plane, that is to say in the Laplace space [7]:

$$\theta_s(0, 0, p) = \frac{\phi}{p} \frac{1 + R_c E S \sqrt{p}}{m_s c_s p + [R_c m_s c_s p + 1] E S \sqrt{p}}. \quad (8)$$

And after inversion at 'long times':

$$T_s(0, 0, t) = \phi \left[ R_c - \frac{m_s c_s}{E^2 S^2} \right] + \frac{2\phi}{E S \sqrt{\pi}} \sqrt{t}. \quad (9)$$

Thus, a first approximate value of the thermal effusivity  $E$  can be estimated from the mean slope  $\alpha$  of the experimental curve  $T_s(0, 0, t) = f(\sqrt{t})$  determined by linear regression between two times  $t_0$  and  $t_1$  such that this curve is considered to be a straight line in this time interval. The obtained result is then

used as an initial value for an estimation of the parameters  $E$ ,  $R_c$  and  $m_s c_s$  using Newton's method. This method is used to minimize the sum of the quadratic errors between the experimental curve  $T_s(0, 0, t)$  and the curve representing the values calculated by inversion of formula (8) using the Stiefest method. It can also be pointed out that a light shift of the thermocouple compared to the centre of the hot strip will not affect the measuring accuracy. It will simply modify the time during which one will be able to make the assumption of unidirectional transfer.

### 4.2. Thermal conductivity estimation

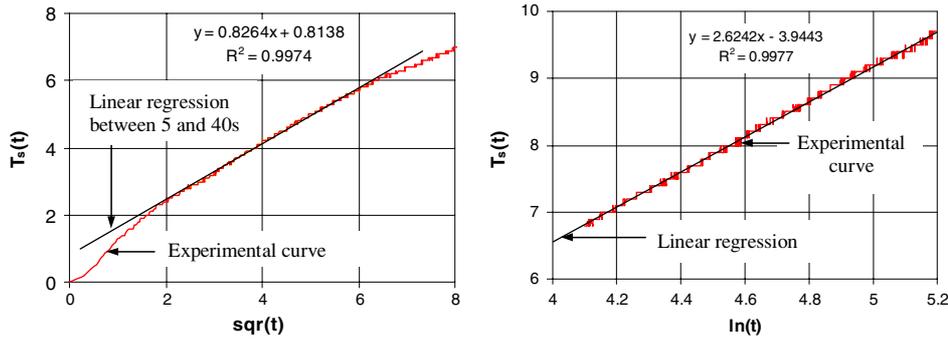
The temperature of an infinite hot strip tends asymptotically towards that of the hot wire as shown by Ladevie [9]. By neglecting the effect of the inertia of the probe, the Laplace transform of the temperature of the hot wire with radius  $r_{hw}$  and length  $L_{hw}$  is

$$\theta_{hw}(p) = \frac{\phi}{p} \left[ R_c + \frac{K_0(qr_{hw})}{2\pi\lambda L_{hw} q r_{hw} K_1(qr_{hw})} \right]. \quad (10)$$

For long times, it is possible to linearize the temperature  $T_s(t)$  in the form:

$$T_s(t) = \frac{\phi}{4\pi\lambda L_{hw}} \ln(t) + \phi \left[ R_c - \frac{\ln\left(\frac{r_{hw}}{\sqrt{a}}\right)}{2\pi\lambda L_{hw}} + \frac{0.577}{4\pi\lambda L_{hw}} \right]. \quad (11)$$

However, the hot strip behaves as a hot wire after a very long time interval so that this asymptotic behaviour cannot



**Figure 3.** Linear regressions (used for a first estimation of  $E$  and  $\lambda$ ) of an experimental curve  $T_s = f(t)$  obtained for polyethylene with a hot strip of 1.2 cm width.

be used to estimate the thermal conductivity in a relatively short time as intended. If the slope of the curve  $T_s(0, 0, t) = f[\ln(t)]$  varies for the hot strip, its variation is sufficiently slow to consider it as a constant on reduced time intervals. A first approximate value  $\lambda_m$  of thermal conductivity  $\lambda$  is estimable from the slope  $\beta \approx \frac{\phi}{4\pi\lambda 2\ell}$  of the experimental curve  $T_s(0, 0, t) = f[\ln(t)]$  determined by linear regression between 120 and 180 s. The obtained result is then used as an initial value for an estimation of the value of the thermal conductivity  $\lambda$ . A dichotomic method is used to minimize the sum of the quadratic errors between the experimental curve  $T_s(t)$  and the curve representing the values computed by using formulae (4)–(6) of the complete model. In these formulae, the values of  $E$ ,  $R_c$  and  $m_s c_s$  estimated between 0 and  $t_1$  are considered as data.

### 5. Optimization of hot strip dimensions

The half-width  $b$  of the hot strip must be such that the time during which the transfer is unidirectional at the centre of heating resistance is sufficiently high to allow a satisfactory estimation of the effusivity  $E$ . One can estimate an upper value of this time in the case of a hot strip of null mass and with a null contact resistance by using formula (7). The time during which the difference between the temperature at the centre of the hot strip and the temperature of the hot plane subjected to the same heat flux density is less than 1% can be deduced from the relation:

$$\operatorname{erf}(X) - \frac{X}{\sqrt{\pi}} \operatorname{Ei}(-X^2) = 0.99 \quad (12)$$

with

$$X = \frac{b}{2\sqrt{at}}.$$

The solution of equation (12) is  $X = 1.31$ . The value  $2b$  of the hot strip width such that the temperature of the centre did not vary more than 1% compared to that of the hot plane at the end of time  $t_1$  can be calculated by

$$2b = 4X\sqrt{at_1}. \quad (13)$$

It is remarkable that this width only depends on the thermal diffusivity of the material. This formula also makes it possible to consider the upper limit  $t_1$  of the estimation interval of the thermal effusivity  $E$  if one has a first estimation of  $\lambda$  and  $E$ . The length  $2\ell$  of the hot strip must be selected so that the heat transfer remains bidirectional until time  $t_2$ . An upper value

of this length can be estimated using formula (13) in which  $t_1$  is replaced by  $t_2$  and considering that it is the necessary dimension so that the transfer remains assimilable to that of a hot plane during  $t_2$  with the following relation (14):

$$2\ell = 2b\sqrt{\frac{t_2}{t_1}}. \quad (14)$$

### 6. Experimental results and discussion

An experimental study has been carried out using an electrical resistance MINCO HK913P having the following characteristics: electrical resistance = 61.1  $\Omega$ , heating surface = 5.4 cm  $\times$  1.2 cm and thickness 0.015 cm. A thermocouple  $K$  made of 0.02 cm diameter wires has been stuck on the hot strip with an adhesive strip. A flux step was applied to the hot strip disposed between two samples of the material to be characterized. The thermocouple temperature was recorded for 180 s with a time step of 0.1 s by an ALMEMO 2290-4 apparatus whose resolution is 0.1  $^\circ\text{C}$ . Three materials were tested: PVC, PCTFE and polyethylene, assumed to be isotropic. The typical sample area was 4 cm  $\times$  6 cm and the typical thickness was 1.5 cm. They were selected for their low diffusivity which ensures behaviour of hot plane type for a sufficiently long time  $t_1$  taking into account the width of the hot strip. The values of the density were measured with the mercury volume meter described by Talla *et al* [11]. Their calorific capacity was measured in a differential scanning calorimeter SETARAM TG-DSC111. Five measurements were carried out on each sample. An example of an experimental curve is presented in figure 3 in the form  $T_s = f(\sqrt{t})$  between 0 and 60 s and  $T_s = f[\ln(t)]$  between 60 and 180 s. This curve was obtained with the hot strip previously described submitted to an electrical tension step of 7.34 V and placed between two polyethylene samples.

It can be noted that between 5 and 40 s, the curve  $T_s = f[\ln(t)]$  is almost a straight line. This allows us to estimate a first value of the thermal effusivity:  $E = 910.0 \text{ J m}^{-2} \text{ }^\circ\text{C}^{-1} \text{ s}^{-1/2}$ . It can also be noted that between 60 and 120 s the curve  $T_s = f[\ln(t)]$  is almost a straight line. This allows us to estimate a first value of the thermal conductivity:  $\lambda = 0.495 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ . An estimation of the parameters  $E$ ,  $R_c$  and  $m_s c_s$  by the Newton method (integrated in the solver of a widely used spreadsheet) applied to the complete model led

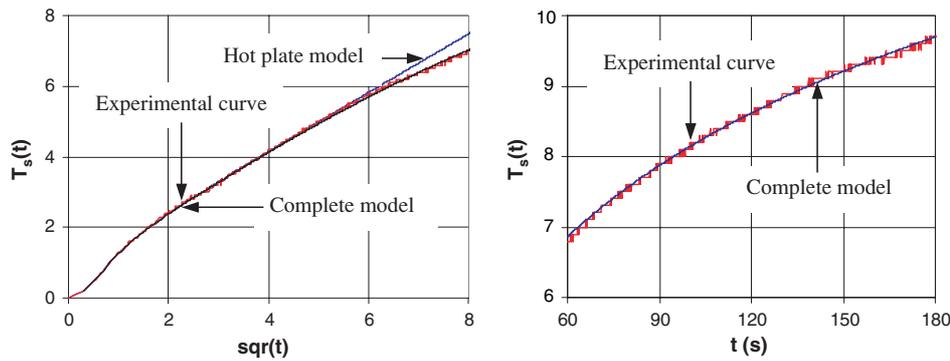


Figure 4. Experimental and simulated curves for polyethylene with a hot strip of 1.2 cm width.

Table 1. Experimental results.

	PVC	PCTFE	Polyethylene
Time interval			
For the estimation of $E$	0–30 s	0–40 s	0–25 s
For the estimation of $\lambda$	30–180 s	40–180 s	25–180 s
$E$ ( $\text{J m}^{-2} \text{ } ^\circ\text{C}^{-1} \text{ s}^{-1/2}$ )	530.4	692.8	888.6
Standard deviation (%)	(1.7)	(1.4)	(3.4)
$\lambda$ ( $\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$ )	0.214	0.228	0.456
Standard deviation (%)	(4.2)	(3.0)	(3.6)
Hot strip $\rho c = E^2/\lambda$ ( $\text{kJ m}^{-3} \text{ } ^\circ\text{C}^{-1}$ )	1315	2105	1732
$R_c$ ( $\text{W } ^\circ\text{C}^{-1}$ )	3.1	2.58	3.18
Standard deviation (%)	(23.1)	(13.0)	(29.3)
$m_s c_s$ ( $\text{J } ^\circ\text{C}^{-1}$ )	0.249	0.172	0.208
Standard deviation (%)	(23.0)	(13.5)	(21.0)
Volume meter $\rho_0$ ( $\text{kg m}^{-3}$ )	1459	1782	929
DSC $c_0$ ( $\text{kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ )	930	1112	1830
$\rho_0 c_0$ ( $\text{kJ kg}^{-3} \text{ } ^\circ\text{C}^{-1}$ )	1357	1981	1700
Deviations $100 \frac{\Delta(\rho_0 c_0 - \rho c)}{\rho c}$ (%)	3.1	6.2	1.9

to the following result:  $E = 934.0 \text{ J m}^{-2} \text{ } ^\circ\text{C}^{-1} \text{ s}^{-1/2}$ ;  $R_c S = 1.7 \times 10^{-3} \text{ } ^\circ\text{C m}^{-2} \text{ W}^{-1}$  and  $m_s c_s = 0.19 \text{ J } ^\circ\text{C}^{-1}$ .

The complete model was then used with these values of  $E$ ,  $R_c$  and  $m_s c_s$  now considered as data. An estimation by a simple dichotomic method with minimization of the standard deviations between 0 and 180 s leads to the value  $\lambda = 0.461 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1}$ . The experimental curves as well as the curves calculated by the complete models of the hot plane and of the hot strip are represented in figure 4. A very good agreement between the curves is observed. The syntheses of the results obtained by the different methods are presented in table 1.

The estimated values of  $m_s c_s$  are not strictly the same for each test. It may be explained by the existence of a temperature gradient within the heater along the  $y$ -axis and by the fact that the temperature is measured on the external face of the heater. The mean heater temperature is thus underestimated leading to an overestimated value of  $m_s c_s$ . The difference between the mean value of the heater temperature and the measured one depends on the heat flux intensity and on the thermal contact resistance that are not constant for each test. This may explain the observed differences and prevent considering the same value of  $m_s c_s$  for each test.

The mean values of the thermal capacitance  $\rho c$  estimated by the method of the hot strip on one hand and by measurement

of the density by a volume meter coupled with measurement of the heat capacity by a differential scanning calorimeter on the other hand are in good agreement (error < 6.2%) for each material. The standard deviations of the estimated values of the thermal effusivity  $E$  and of the thermal conductivity  $\lambda$  are also quite low (< 4.2%).

## 7. Conclusion

This study shows that it is possible to estimate simultaneously with a good precision the thermal effusivity and thermal conductivity of a material using a simple inexpensive probe: a thermocouple fixed by an adhesive tape on an electrical resistance. The specificity of the method is also the simplicity of the parameter estimation method carried out in a very common spreadsheet, using its integrated solver. The method was validated on low diffusivity materials:  $a < 2 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ , which is a typical value for plastics and woods according to Edwards [12]. It must now be tested with a broader hot strip on higher diffusivity materials:  $2 \times 10^{-7} \text{ m}^2 \text{ s}^{-1} < a < 10^{-6} \text{ m}^2 \text{ s}^{-1}$ , which is a typical value for buildings materials [12]. The study of sensitivity also shows that its application will require a broader strip and a longer time measurement in the case of materials of very low

thermal effusivity ( $E < 100 \text{ J m}^{-2} \text{ }^\circ\text{C}^{-1} \text{ s}^{-1/2}$ ). For these materials, an improvement of the proposed method could be the replacement of the flux step applied to the hot strip by other solicitation types: periodic or random flux for example. This possible improvement will be studied later.

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