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A quadrupolar complete model of the hot disc

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Abstract
The hot disc method is a transient plane source method used for the estimation of the thermal conductivity and diffusivity of solid materials. A complete model based on the thermal quadrupoles formalism has been developed to represent the hot disc temperature variation. This model takes into account both the thermal contact resistance between the solid to be characterized and the hot disc and the thermal inertia of the hot disc. It makes it possible to realize the parameters estimation on all the recorded temperature measurements. This model is used to highlight the estimation uncertainty due to approximations in the heat transfer model.

Keywords: transient hot disc, thermal quadrupoles, model, mass sensitivity, conductivity, diffusivity, parameter estimation

(Some figures in this article are in colour only in the electronic version)

Nomenclature

a	thermal diffusivity (m^2 s^{-1})
c	specific heat of the hot disc (J kg^{-1} $°C^{-1}$)
D	characteristic function of the hot disc
dt	recording time step (s)
e	sample thickness (m)
I	transfer function in Laplace space
I_0	modified Bessel function of the first kind of order zero
J_1	Bessel function of the first kind of order one
m	mass of the hot disc (kg)
n	number of concentric resistive rings
p	Laplace parameter
r_0	hot disc radius (m)
R_c	thermal contact resistance (°C W^{-1})
S	hot disc area (m^2)
T	hot disc mean temperature (°C)
V_j	coefficients of the Stehfest algorithm
X	sensitivity matrix of T
Z_c	constriction impedance
Z_{cs}	approximate value of the constriction impedance
θ	Laplace transform of the hot disc mean temperature
θ_{se}	Laplace transform of the mean temperature of the sample surface
ϕ_0	heat flux dissipated in the hot disc (W)
Φ	Laplace transform of the hot disc heat flux
Φ_{se}	Laplace transform of the heat flux on the sample surface
λ	thermal conductivity (W m^{-1} $°C^{-1}$)
ρ	hot disc density
τ	dimensionless time

Subscripts

n	number of double spirals
∞	hot disc with uniform heat flux

1. Introduction

The hot disc method is an experimental method designed for the estimation of the thermal conductivity and thermal diffusivity of solid samples. This transient plate source method is based on the heating of a plane double resistive spiral sandwiched between two samples of the material to be characterized. The recording of the mean temperature of the heating element with an estimation parameters method applied to a theoretical model makes it possible to estimate thermal conductivity and thermal diffusivity of the samples from a single experiment. This method was studied and used by many authors (Gustafsson 1991, Bohac *et al* 2000, Malinaric 2004, He 2005).

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The setting up of this method still raises some questions:

- The theoretical model of the hot disc temperature evolution was published in the form of an infinite integral (Gustafsson 1991, He 2005) without specifying, to our knowledge, by which mathematical function this integral is represented for use. It can be noted that the model considers the resistive spiral as concentric resistive rings with negligible width.
- The proposed simplified model does not take into account the influence of the hot disc inertia. During the parameter estimation process, the first points of the recorded temperature (corresponding to times lower than a certain time t_1) are not taken into account. The time t_1 is empirically estimated (Bohac *et al* 2000): several successive estimations on intervals $[t, t_2]$ are realized, where t_2 is a constant and selected so that the dimensionless time $\tau_2 = \sqrt{\frac{at_2}{r^2}}$ is equal to 1.0, where a is the thermal diffusivity of the sample and r is the radius of the hot disc probe. The time t is progressively increased by starting from $t = 0$ s. Up to a certain time t_1, the estimated values of the parameters vary very weakly and the interval $[t_1, t_2]$ is then selected for the estimation.

The aim of this study is to establish a quadrupolar model of heat transfer during a hot disc experience in order to estimate the temperature sensitivity to the following parameters: thermal conductivity λ, thermal diffusivity a, thermal contact resistance R_c and hot disc thermal capacity mc.

Furthermore, this model must make possible the estimation of all these parameters. The interest of such a complete model is to take into account all experimental points without empirically removing a certain number of them.

Another purpose of this work is to evaluate the estimation errors on thermal conductivity and diffusivity due to the imperfection of the model used for heat transfer representation in a hot disc/sample system.

2. Modelling

2.1. Simplified model

Considering a hot disc assimilated to n uniformly spaced concentric thin rings of a resistive material, it has been shown that the mean temperature T of the disc submitted to a total output power φ_0 can be written as (Gustafsson 1991)

$$T(t) = \frac{\varphi_0}{\pi^{3/2} r_0 \lambda} D_n(\tau) \tag{1}$$

with $\tau = \sqrt{\frac{at}{r_0^2}}$ where a is the thermal diffusivity of the sample, λ is the thermal conductivity of the sample and r_0 is the hot disc radius and

$$D_n(\tau) = [n(n+1)]^{-2} \int_0^{\tau} \mathrm{d}\sigma\, \sigma^{-2} \times \left[\sum_{l=1}^{n} l \sum_{k=1}^{n} k \exp\left(-\frac{l^2+k^2}{4n^2\sigma^2}\right) I_0\left(\frac{lk}{2n^2\sigma^2}\right)\right] \tag{2}$$

where n is the number of concentric resistive rings of the hot disc, I_0 is the modified Bessel function of the first kind of order zero.

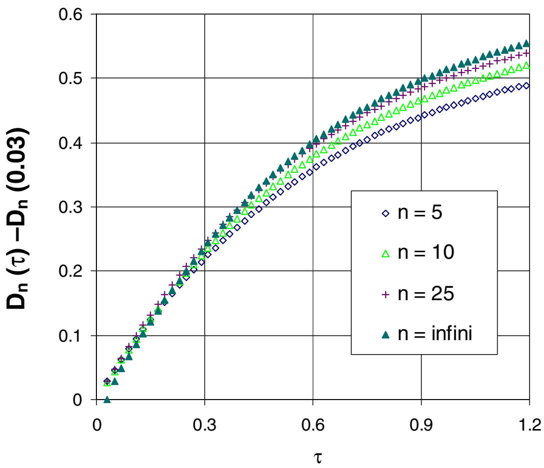

Figure 1. Functions $D_n(\tau) - D_n(0.03)$.

If the space between the resistive rings becomes very thin, this solution tends towards

$$D_\infty(\tau) = \int_0^{\tau} \mathrm{d}\sigma\, \sigma^{-2} \int_0^1 v\, \mathrm{d}v \int_0^1 u\, \mathrm{d}u \times \exp\left(-\frac{u^2+v^2}{4\sigma^2}\right) I_0\left(\frac{uv}{2\sigma^2}\right). \tag{3}$$

The numerical calculation of $D_n(\tau)$ and $D_\infty(\tau)$ is not possible near zero (for $\tau < 0.03$) because relations (2) and (3) lead to an infinite limit when τ tends towards zero.

By assuming the hot disc as concentric resistive rings of infinitely small width, the relations (2) and (3) do not allow a good representation of hot disc temperature variation at very short times. Indeed, one can see that the resistive rings width does not appear in these two relations.

Thus, we calculate $D_n(\tau) - D_n(0.03)$ and $D_\infty(\tau) - D_\infty(0.03)$ and represent them by a six degree polynomial as $P_n(\tau) = b_0 + b_1\tau + b_2\tau^2 + b_3\tau^3 + b_4\tau^4 + b_5\tau^5 + b_6\tau^6$

So, one can deduce that $D_n(\tau) = P_n(\tau) + D_n(0.03)$ and as $D_n(0) = 0$:

$$D_n(\tau) = D_n(0.03) + b_0 + b_1\tau + b_2\tau^2 + b_3\tau^3 + b_4\tau^4 + b_5\tau^5 + b_6\tau^6.$$

Table 1 and figure 1 present, respectively, the coefficients of each polynomial and the representative curves of the corresponding functions.

It can be noted that the coefficient b_1 tends towards 1 for an infinite number of resistive rings (uniform flux). Then, at short time, we have $D_\infty(\tau) \approx D_\infty(0.03) + b_0 + \tau$ and so:

$$T(t) = \frac{\varphi_0[D_\infty(0.03) + b_0 + \tau]}{\pi^{3/2} r_0 \lambda} = \frac{\varphi_0\left[D_\infty(0.03) + b_0 + \sqrt{\frac{at}{r_0^2}}\right]}{\pi^{3/2} r_0 \lambda} = \frac{\varphi_0}{\pi r_0^2 \sqrt{\pi} E}\sqrt{t} + \frac{\varphi_0[D_\infty(0.03) + b_0]}{\pi^{3/2} r_0 \lambda}.$$

Considering a uniform flux, we must find the hot plate equation at short time. Thus, we deduce that for an infinite number of resistive rings $b_0 = -D_\infty(0.03)$.

With a finite number n of concentric resistive rings, function $D_n(\tau)$ is known only for $\tau > 0.03$:

$$D_n(\tau) = c_0 + b_1\tau + b_2\tau^2 + b_3\tau^3 + b_4\tau^4 + b_5\tau^5 + b_6\tau^6,$$

Table 1. Coefficient values of polynomials $D_n(\tau) - D_n(0.03)$.

	$n = 5$	$n = 8$	$n = 10$	$n = 16$	$n = 25$	$n = 32$	$n = \infty$
b_1	0.9596	0.905 57	0.914 1	0.938 6	0.956 4	0.963 8	0.996 7
b_2	−1.0872	−0.539 3	−0.494 2	−0.481 2	−0.483 7	−0.486 1	−0.498 6
b_3	1.5965	0.121 1	−0.032 90	−0.132 2	−0.170 3	−0.182 8	−0.225 7
b_4	−1.9920	−0.081 65	0.120 6	0.256 1	0.312 3	0.331 9	0.400 6
b_5	1.3271	0.121 5	−0.003 580	−0.084 52	−0.117 8	−0.129 5	−0.170 8
b_6	−0.3437	−0.046 20	−0.016 10	0.002 431	0.009 822	0.012 43	−0.021 65

where c_0 is an unknown constant which does not affect the shape of the curve $D_n(\tau)$, it merely causes a constant time lag on $T(t)$ proportional to φ_0 and inversely proportional to λ.

This 'ideal' solution has been established for a probe with negligible mass and a contact resistance R_c between the probe and the sample.

For taking contact resistance R_c into account, it has been assumed that after a certain time the effect of the contact resistance consists merely of a constant shift of the temperature compared to the ideal model (Gustafsson 1991). With his hypothesis one can write

$$T(t) = \frac{\varphi_0}{\pi^{3/2} r_0 \lambda} D(\tau) + R_c \varphi_0. \tag{4}$$

This solution is valid on the assumption that the sample can be regarded as a semi-infinite medium during the time of measurement, i.e. if $d \geqslant \sqrt{2at_f}$, where d is the smallest distance from the probe to the external surface of the sample and t_f is the duration of the experiment (Log and Gustafsson 1995).

This simplified modelling does not allow the estimation of the time after which the influence of contact resistance and probe inertia are perfectly represented by a shift on the temperature, as indicated in relation (4).

2.2. Complete model

Contact resistance and hot disc mass are taken equal to zero. For a hot disc of radius r_o placed between two semi-infinite samples and subjected on its surface to a homogeneous heat flux $2\varphi_0$, it has been shown that the Laplace transform of the mean temperature elevation is (Maillet *et al* 2000)

$$\theta(p) = Z_c \frac{\varphi_0}{p} = \frac{\varphi_0}{p} \frac{2}{\pi r_0 \lambda} \int_0^\infty \frac{[J_1(\varepsilon)]^2}{\varepsilon \sqrt{\varepsilon^2 + \frac{p r_0^2}{a}}} \, \mathrm{d}\varepsilon. \tag{5}$$

An approximated value Z_{cs} of the constriction impedance Z_c has been proposed as

$$Z_{cs} = \frac{8}{3\pi^2 \lambda r_0 \left(1 + \frac{8}{3\pi} \sqrt{\frac{p r_0^2}{a}}\right)}. \tag{6}$$

The deviations between Z_c and Z_{cs} are not negligible since Z_{cs} values lead to satisfactory asymptotic representations of Z_c.

Thus, a more precise Z_c expression has been researched by using an asymptotic development of the Bessel function J_1 to infinity. Numerical calculation leads to the following result:

$$Z_c = \frac{2}{\pi r_0 \lambda} I(p) \tag{7}$$

with

$$I(p) = \frac{1 + 0.382\,48\sqrt{p^*}}{2.356\,06 + 2.286\,82\sqrt{p^*} + 0.764\,96 p^*}$$

$$\text{where} \quad p^* = \frac{p r_0^2}{a}. \tag{8}$$

Table 2. Coefficient values of polynomial functions $I_n(p)$.

n	5	8	10	16	25	32
d_1	6.8559	0.7223	0.6458	0.5999	0.5759	0.5631
e_0	10.3827	5.2843	5.1859	5.0855	5.0134	4.9796
e_1	29.4882	6.0943	5.7153	5.3948	5.2233	5.1490
e_2	30.2988	3.2558	2.8785	2.6027	2.4525	2.3806

The calculation of the $D_\infty(\tau)$ function is done by inversion of the previous expression with the Stehfest method (Stehfest 1970) leading to

$$D_\infty(\tau) = \frac{\pi^{3/2} \lambda r_0}{\varphi_0} L^{-1}[\theta(p)]. \tag{9}$$

The relative deviations between $D_\infty(\tau)$ values, calculated with formulae (3) and (9), are lower than 1% for $\tau < 1.2$, showing the equivalence of the two approaches. The interest of relation (8) is to make it possible to take into account the probe inertia and the contact resistance in a complete quadrupolar model.

From $D_n(\tau)$ functions, $I_n(p)$ functions can be determined thanks to the relation

$$\frac{\varphi_0}{\pi^{3/2} r_0 \lambda} D_n(\tau) = L^{-1}\left[\frac{\varphi_0}{p} \frac{2}{\pi r_0 \lambda} I_n(p)\right], \tag{10}$$

i.e.,

$$\frac{1}{2\sqrt{\pi}} D_n(\tau) = L^{-1}\left[\frac{1}{p} I_n(p)\right].$$

These functions have been researched in the form

$$I_n(p) = c_0 + \frac{1 + d_1\sqrt{p}}{e_0 + e_1\sqrt{p} + e_2 p}. \tag{11}$$

The coefficients obtained for different values of n are reported in table 2.

The c_o coefficient is equal to $\frac{b_0}{2\sqrt{\pi}}$ and to 0 for an infinite number of resistive rings.

The deviation between $D_n(\tau)$ values calculated with the polynomial whose coefficients are given in table 1, and those estimated from relation (10), whose coefficients are given in table 2, is below 0.5% for $\tau > 0.05$.

For a hot disc with a thermal capacitance mc and a contact resistance R_c between the hot disc and the sample, one can write the following matrix equation (Maillet *et al* 2000):

$$\begin{bmatrix} \theta(p) \\ \Phi(p) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ mcp & 1 \end{bmatrix} \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{se}(0, p) \\ \theta_{se}(0, p)/Z_c \end{bmatrix}$$

where θ is the Laplace transform of the hot disc mean temperature, Φ is the Laplace transform of the hot disc flux, θ_{se} is the Laplace transform of the mean temperature of the sample surface and Φ_{se} is the Laplace transform of the flux on the sample surface in contact with the hot disc.

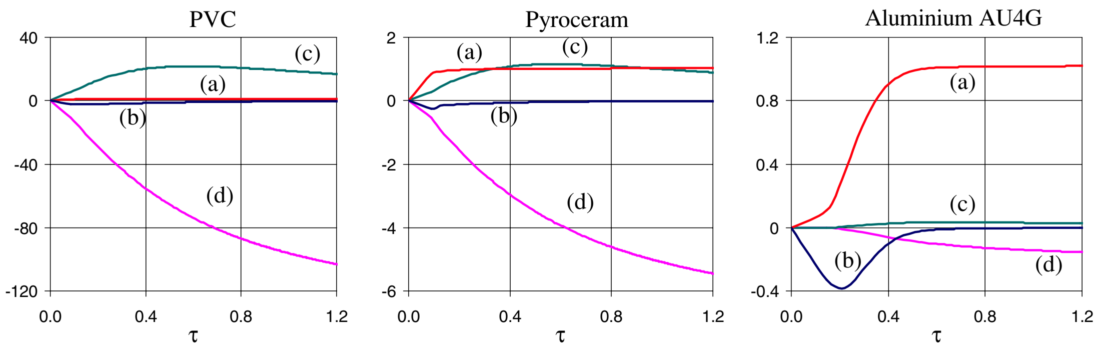

Figure 2. Reduced sensitivities of the hot disc temperature to several parameters: (*a*) *Rc*; (*b*) *mc*; (*c*) *a*; (*d*) λ for different materials—PVC: $\lambda = 0.21$ W m^{-1} °C^{-1}, $a = 1.7 \times 10^{-7}$ m^2 s^{-1}; pyroceram: $\lambda = 3.98$ W m^{-1} °C^{-1}, $a = 1.89 \times 10^{-6}$ m^2 s^{-1}; aluminium AU4G: $\lambda = 138.5$ W m^{-1} °C^{-1}, $a = 5.88 \times 10^{-5}$ m^2 s^{-1}.

Table 3. Estimated parameter values for five tests on titanium TA6V.

	$r_0 = 3.189$ mm, $n = 8$				
Test	a (m^2 s^{-1})	λ (W m^{-1} °C^{-1})	ρc (J kg^{-1} °C^{-1})	R_c (°C W^{-1})	mc (J °C^{-1})
1	2.60×10^{-6}	6.46	2.48×10^6	4.70	0.0037
2	2.89×10^{-6}	6.64	2.30×10^6	5.34	0.0030
3	2.54×10^{-6}	6.50	2.56×10^6	5.10	0.0027
4	2.71×10^{-6}	6.41	2.37×10^6	5.29	0.0033
5	2.69×10^{-6}	6.61	2.46×10^6	5.12	0.0044
Mean	2.69×10^{-6}	6.52	2.43×10^6	5.11	0.0034
Mean deviation (%)	4.4	1.3	4.2	4.4	17.3

One can deduce that

$$\theta(p) = \frac{\varphi_0}{p} \frac{Z_c + Rc}{Z_c mcp + mcRp + 1} \qquad (12)$$

with $Z_c = \frac{2}{\pi r_0 \lambda} I_n(p)$

3. Sensitivity analysis

Relation (12) allows the reduced sensitivity calculation of $T(t)$ for the different parameters k_i by $k_i \frac{\partial T}{\partial k_i} = 1000\,[T(1,001k_i) - T(k_i)]$ where $T(t) = L^{-1}[\theta(p)]$ is calculated by relation (12).

This reduced sensitivity multiplied by 0.01 represents the absolute variation of $T(t)$ for a relative variation of 1% of the parameter k_i from its initial value.

Relations (11) and (12) have been used to calculate the reduced sensitivities of $T(t)$ to thermal diffusivity a, to thermal conductivity λ, to probe thermal capacity and to the contact resistance R_c. The following experimental conditions have been considered in calculations:

- Hot disc in Kapton; thickness = 0.2 mm, radius, $r_0 = 6.401$ mm, the resistive spiral is assimilated to $n = 16$ concentric resistive rings; thermal properties of Kapton: heat capacity = 1.09×10^6 J kg^{-1} °C^{-1}, density = 1420 kg m^{-3} (Lua and Su 2006),
- Recording of 200 experimental points with a minimum time step of 0.015 s.

Reduced sensitivities obtained for different materials and calculated for $R_c = 1.56$ °C W^{-1}, that is to say $S\,R_c = 2 \times 10^{-3}$ °C m^2 W^{-1}, are plotted in figure 2. It can be noted that sensitivities to the contact resistance R_c and to the hot disc thermal capacity mc become constant for $\tau > 0.3$. Thus, in this example, a simplified model that does not take into account the influence of Rc and mc can be used for parameter estimation only for $\tau > 0.3$ which is in conformity with the estimation interval recommended: $\tau \in [0.3, 1.0]$ (Bohac *et al* 2000). This minimal value $\tau_{\min}$ becomes equal to 0.5 for materials with strong diffusivity as shown in the graph in figure 2 corresponding to aluminium AU4G.

The thermal diffusivity sensitivity reaches a maximum for $\tau = 0.5$ before decreasing whereas the thermal conductivity sensitivity is still growing continuously.

While the thermal diffusivity a and the thermal conductivity λ increase, one can note the following:

- Sensitivities to the thermal conductivity λ and to the thermal diffusivity a strongly decrease. In particular, the sensitivity to diffusivity becomes very low for highly conductive materials.
- The sensitivities to the contact resistance R_c and to the hot disc thermal capacity mc become constant only for τ values higher than 0.5.

The first point implies that the hot disc method cannot make it possible to estimate high thermal diffusivities with precision. This is especially true when the parameter estimation is based on a simplified model that does not take into account the transitory effects of the thermal contact resistance R_c and of the hot disc thermal capacitance mc.

The standard deviations of estimation error, due to the measurement noise on T, have been estimated by taking the root of the diagonal terms of the matrix $\sigma T^2[X^t X]^{-1}$, where X is the sensitivity matrix (Beck and Arnold 1977). The standard deviation σT of error measurement on $T(t)$ has been considered

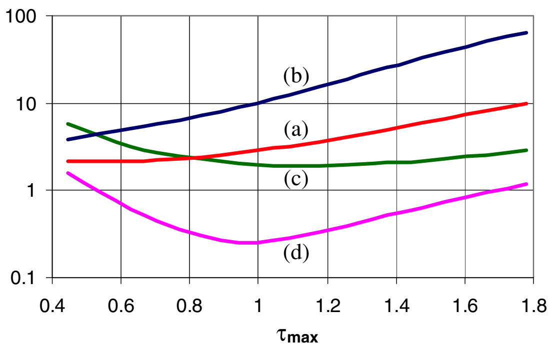

Figure 3. Relative standard deviations (%) of estimation errors in different parameters versus τ_{max}. (*a*) *Rc*; (*b*) *mc*; (*c*) *a*; (*d*) λ.

constant and equal to 0.01 °C, which is the average value observed during experimental records.

Figure 3 represents relative standard deviation values (in %) of the estimated parameters versus τ_{max} (τ value of the last recorded point) for a 200 point temperature recording on titanium TA6V. It can be noted that the standard deviation in thermal diffusivity a and in thermal conductivity λ is minimum for $\tau_{max} = 1.0$. This value is in conformity with the analysis of sensitivities curves (linear relation between the reduced sensitivities to λ and to a for $\tau > 1.0$) and identical to that estimated by other authors (Bohac *et al* 2000).

Moreover, in this case, it is noticeable that the standard deviation in thermal diffusivity is about 3–5 times higher than the standard deviation in thermal conductivity λ. This result is also in conformity with the analysis of the sensitivities curves and with the previous studies (Log and Gustafsson 1995).

4. Experimental results and discussion

A first series of five measurements has been carried out with a hot disc of 3.189 mm radius (value given by the manufacturer) with $n = 8$ concentric rings.

The tested material was titanium TA6V whose properties were measured by different methods:

- Hot plate method with two temperature measurements: $\lambda = 6.61$ W m^{-1} °C^{-1} and $a = 2.70 \times 10^{-6}$ m^2 s^{-1}.
- Flash method: $a = 2.75 \times 10^{-6}$ m^2 s^{-1}.
- Differential calorimeter: volume heat capacity $= 2.32 \times 10^6$ J °C^{-1}.

The theoretical curve $T(t)$ is calculated by applying the Stehfest method to relation (12) with $n = 8$. The Newton method is used to estimate the values of the parameters λ, a, mc and R_c minimizing the sum of the quadratic error between the theoretical and experimental curves. All the recorded values (200) of $T(t)$ have been taken into account in this sum.

In the model, it has been shown that the coefficient c_0 involved in the expression (11) of $I_n(p)$ only produces a constant shift of the curve $T(t)$. It has been verified that its value does not have any influence on the optimal solution, thus it has been taken equal to zero. The obtained results for the five tests are reported in table 2.

The obtained average values of thermal diffusivity a and of thermal conductivity λ are very close (deviation lower than 1.3%) to those measured by hot plate and flash methods.

The comparison of the standard deviations observed in the various parameters produces a result in conformity with figure 3. The value of the volume heat capacity differs by about 4.7% from the one measured by differential calorimeter which is acceptable.

One can find in figure 4 an example of an experimental curve obtained on titanium TA6V under the following conditions: $\varphi_0 = 0.7$ W, recording of 200 points with a time step d$t = 0.027$ s.

The residues of the estimation that are the differences between the experimental and the modelled values of $T(t)$ are also plotted in figure 4. These values are low, however higher values are observed for the first points. This can be explained by the fact that the variation in temperature is very fast at short times and that a small uncertainty in time measurement can cause a significant variation in temperature.

If the five first points are excluded, the standard deviation of the differences between the theoretical curve and the experimental one is about 0.0035 °C.

In the studied model, the effects of the conductive transfer within the Kapton located between the resistive rings are neglected. This conductive transfer will tend to bring the probe behaviour closer to that of a probe with a uniform flux corresponding to an infinite number of spirals. We have taken back the parameter estimation of the test whose results are presented in figure 4 by using function $I_n(p)$ corresponding to n infinite. The following results are obtained:

- $n = 8$: $\lambda = 6.46$ W m^{-1} °C^{-1} and $a = 2.60$ m^2 s^{-1}
- $n = \infty$: $\lambda = 6.81$ W m^{-1} °C^{-1} and $a = 2.34$ m^2 s^{-1}.

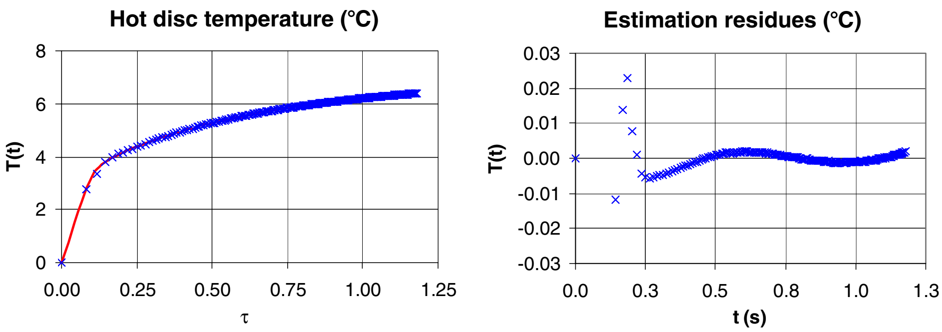

Figure 4. Hot disc temperature and estimation residues for a test on titanium TA6V.

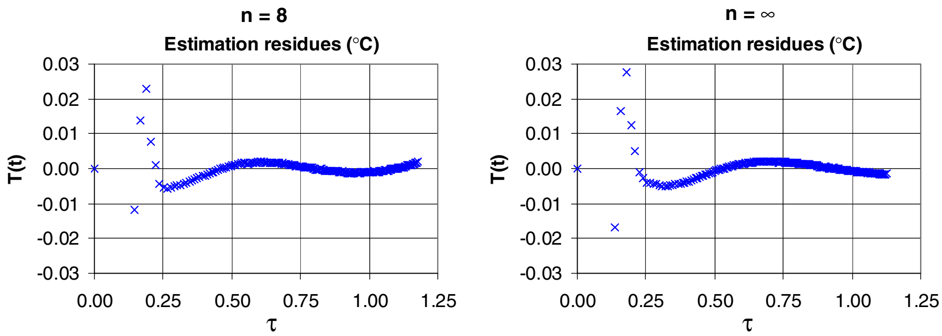

Figure 5. Estimation residues obtained with resistive ring numbers $n = 8$ and $n = \infty$.

The differences between the estimated values with the two models ($n = 8$ and $n = \infty$) are, respectively, 5% for the thermal conductivity and 10% for the thermal diffusivity. The residues of estimations (previously defined) are similar for the two models as shown in figure 5, so that it is difficult to say that one estimation is more precise than the other.

The effective behaviour of the hot disc is between these two extreme cases. It becomes essential to obtain an experimental identification of function $I_n(p)$ by a measurement on a reference sample in order to reach better precision in thermal properties estimation from a hot disc temperature record.

5. Conclusion

A quadrupolar model of the hot disc temperature taking into account the thermal contact resistance R_c and the probe thermal capacitance mc has been established. This model makes possible a sensitivity study which permits us to highlight the influence of the thermal contact resistance R_c and hot disc thermal capacity mc on the temperature $T(t)$ for τ values lower than $\tau_{\min} = 0.3$. This value of $\tau_{\min}$ increases for very diffusive materials, which makes the estimation of the thermal diffusivity for these materials less accurate.

An experimental study has been carried out on a material characterized by three different methods: Flash method, hot plate method with two temperature measurements and a differential calorimeter device. The parameter estimation has been carried out by minimization of the sum of quadratic deviations between all the points of the experimental curve and the theoretical one using the Newton method. The estimated values of thermal conductivity λ and thermal diffusivity a are very close (maximum variation lower than 5%) to those estimated by other methods, these results thus validate the model and the proposed estimation method.

The purpose of this study also lies in the establishment of an explicit model, not requiring any choice of time interval for parameter estimation, providing an evaluation of the errors in the estimated value and leading to a satisfactory precision.

Furthermore, this study makes it possible to highlight uncertainties in the estimated values due to the model imperfections and shows the need for an experimental identification of the I_n function, which is a hot disc characteristic.

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