A SIMULTANEOUS DETERMINATION OF PERMEABILITY AND KLINKENBERG COEFFICIENT FROM AN UNSTEADY-STATE PULSE-DECAY EXPERIMENT

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ABSTRACT

The unsteady state gas flow experiment is an appealing technique to determine the intrinsic permeability k_l and Klinkenberg gas slippage factor b of tight rock samples. "Pulse-Decay" and "Draw-Down" techniques consist in recording the pressure difference $\Delta P(t)$ at the core plug edges when a pulse of higher pressure initially applied at the inlet of the sample, relaxes through the core. The interpretation of the pressure-decay signal represents a challenge for a reliable estimation of k_l and b. On the basis of a previous analysis where the optimal experimental conditions were defined, the objective of this paper is to present an inverse technique that makes use of a complete model of interpretation. To begin with, we recall the classical method of interpretation based on a simplified model proposed in the literature. To circumvent some of the difficulties inherent to this method, we show how an inverse technique, based on the minimization algorithm of Levenberg - Marquardt (1963), can be used along with a complete flow model to perform the estimation of k_l and b. Uncertainties on experimental parameters such as the volume of the upstream reservoir, dimensional characteristics of the sample and porosity of the material are introduced, the impact of which on the precision of the estimated values of k_l and b is evaluated. For the sake of clarity and to carefully analyze each parameter independently, the method is illustrated on synthetic signals.

INTRODUCTION

Reliable determination of intrinsic permeability, k_l, and Klinkenberg coefficient, b, from laboratory-scale experiments on core plug samples of low permeability media is a challenging task as testified by the concentration of research efforts on the subject over several decades (Bruce *et al.*, 1952; Kaczmarek, 2008). This issue has been recently renewed by the increasing interest on tight rocks, justifying a more careful inspection of experimental conditions and expected precision for the measurement of one phase flow properties of such materials (Jannot *et al.*, 2007). For permeabilities ranging from tens of μDarcies to nDarcies, unsteady-state gas permeability measurement is preferred to steady-state techniques. This allows to circumvent difficulties of the latter associated with i) the precise measurement of extremely small flow rates even at high pressure drops, ii) the time required to reach a steady flow that roughly varies with the square of the sample length and is inversely proportional to the intrinsic permeability when constant pressures applied at the upstream and downstream faces of the sample are considered in a 1D experiment, iii) the necessity of running several measurements each performed at different mean pressure levels (Rushing *et al.*, 2004; Blanchard *et al.*, 2007) to identify k_l and b, leading to an excessively long experimental procedure. Conversely, the unsteady-state experiment is much faster – a single pressure decay signal can be sufficient to identify both k_l and b. In essence, this experiment, often referred to as the pulse-decay technique, consists in recording the relaxation through the sample of a highly pressurized gas confined in a tank (volume V_0) that is opened at the upstream edge of the core at t=0, the downstream being connected to a tank of finite volume (V_1) in the pulse-decay experiment or infinite volume (atmosphere) in the draw-down experiment. Basically, this type of experiment was designed during the 50's (Bruce *et al.*, 1952; Aronofsky, 1954; Wallick and Aronofsky, 1954; Aronofsky *et al.*, 1959) although several variants were proposed since then for i) faster experiments (Jones, 1997), ii) application to partially water saturated samples (Newberg and Arastoopour, 1986; Homand *et al.*, 2004), iii) permeability measurement with an incompressible fluid (Trimmer, 1982; Amaefule *et al.*, 1986), iv) unsteady-state probe permeametry (Jones 1994).

Although pulse-decay or draw-down experiments are fast, their interpretation on the basis of complete unsteady models including compressibility, Klinkenberg effect (and sometimes Forchheimer effect) to determine the sample permeability can not be performed analytically in a simple manner due to the strong non-linearities. However, an abundant literature has been dedicated to the interpretation of pressure signals in such 1D experiments. Assuming negligible Klinkenberg effect and constant mass flow-rate along the sample, approximated solutions to pressure relaxation over time were derived under the form of series (Brace *et al.*, 1968; Hsieh *et al.*, 1981; Neuzil *et al.*, 1981; Chen and Stagg, 1984; Haskett *et al.*, 1988; Dicker and Smits, 1988, Wang and Hart, 1993), error functions (Bourbie and Walls, 1982) or exponential decay (Dana and Skoczylas, 1999; Ivanov *et al.*, 2000). A similar type of approach was adopted for the radial configuration (Gillicz, 1991). Inertial effects were accounted for in a very simplified form to interpret experimental pulse-decay data by Innoncentini *et al.* (2000). Approximated analytical interpretations for the draw-down experiment including Klinkenberg (and Forchheimer) effects were also proposed (Jones, 1972). Analytical solutions to 1D pulse-decay and draw-down pressure signals including Klinkenberg effect but assuming constant mass flow-rate were proposed recently (Kaczmareck, 2008) while the case of radial flow under the same conditions was analyzed by Wu *et al.* (1998).

Despite this large number of references focused on interpretative models, very few have been dedicated to the analysis of the impact of user-adjustable experimental parameters including volumes of upstream and downstream tanks, V_0 and V_1, sample geometrical characteristics, initial upstream pressure, experimental duration *etc.* on the expected precision of k_l and b. Based on the simplifying hypotheses of a constant mass flow-rate along the sample and without Klinkenberg effect, it was concluded that k_l is better estimated when $V_0 = V_1$ (Dicker and Smits, 1988, Jones, 1997). In addition, if porosity is to be estimated, it is preferable to match V_0 and V_1 to the sample pore volume (Wang and Hart, 1993). The analysis proposed in this last reference was completed by that of Escoffier *et al.* (2005) where a short sensitivity analysis on permeability and specific storage coefficient (without Klinkenberg effect) was proposed. A more complete work on sensitivity and uncertainty on k_l and b was reported by Finsterle and Persoff (1997) in which a complete model along with a full inverse technique was employed. Impact of the measurement error on the feasible estimation of Klinkenberg coefficient (and inertial resistance factor) was discussed by Ruth and Kenny (1989). More recently, Jannot et al. (2007) carefully addressed the impact of the different experimental parameters mentioned above on the expected precision on the estimation of k_l and b using a detailed sensitivity analysis with a complete flow model. This analysis yielded fundamental results in terms of the optimal experimental design and showed in particular that :

- the draw-down configuration (infinite V_1) is the optimal one to estimate k_l and b. Due to the fact that the reduced sensitivity of the pressure decay to k_l are sufficiently uncorrelated in such an experiment, these two quantities can be estimated simultaneously from a single pressure decay record;
- the upstream tank volume, V_0, sample diameter, D, and sample length, e, have no significant influence on the precision expected on the estimation of k_l and b. These user-adjustable parameters only influence experimental duration that is shortened when small V_0, small e and large D are chosen. Note that these two last parameters might also be constrained by representativeness of the material;
- the experiment must be as long as possible since the precision expected on the estimated values of k_l and b almost linearly decrease with the amount of decrease of the initial pressure in the upstream tank.

On this basis, our purpose in the present work is to pursue our investigation of the estimation of k_l and b from a draw-down pressure decay signal using inverse modelling. The gain achieved by using this procedure based on the complete flow model with respect to simplified techniques proposed in the literature is outlined. Using synthetic signals where parameters are well controlled, we further analyze the impact of errors in the different parameters on the estimated values of k_l and b using the complete inverse procedure, leading to important concluding recommendations for meaningful estimation of these properties.

PHYSICAL MODEL

In this section, we quickly recall the boundary value problem describing the 1D gas flow along the x axis during a pulse-decay experiment. The initial mass of gas (dynamic viscosity μ) at pressure P_{0i} contained in the upstream tank of volume V_0 flows from t=0 on through the porous sample of diameter D, length e, porosity ε, intrinsic permeability k_l and Klinkenberg coefficient b. The gas is captured at the outlet of the sample in the downstream tank of volume V_1 and initial pressure P_{1i}. Assuming the flow to be isothermal and slow enough to neglect inertial effects while considering ideal gas law, the mass and momentum balance equations can be combined to obtain:

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\alpha}{\sqrt{\phi}} \frac{\partial \phi}{\partial t}, \qquad \alpha = \frac{\varepsilon \mu}{k_l} \tag{1}$$

Here, we have used $\phi = (P + b)^2$ as the new variable along with the apparent permeability of the gas given by $k_l(1 + b/P)$. The associated initial conditions are $\phi(0,0) = (P_{0i} + b)^2$ and $\phi(x,0) = (P_{1i} + b)^2$, x>0. The boundary conditions at the entrance (x=0) and exit (x=e) faces of the sample are $\frac{\partial \phi}{\partial t}(0,t) = \frac{k_l S}{\mu V_0}\left(\sqrt{\phi}\frac{\partial \phi}{\partial x}\right)_{(0,t)}$ and $\frac{\partial \phi}{\partial t}(e,t) = -\frac{k_l S}{\mu V_1}\left(\sqrt{\phi}\frac{\partial \phi}{\partial x}\right)_{(e,t)}$ where S is the section of the sample. While taking $V_1 \to \infty$, the last boundary condition reduces to P(e,t)=Cst=P_{1i} *i.e.* $\phi(e,t) = (P_{1i} + b)^2$ which is the proper one for the draw-down experiment, indicating that this configuration can be considered simply as a particular situation of the pulse-decay. Following conclusions achieved in our previous investigation (Jannot *et al.*, 2007) and briefly recalled in the introduction, we only consider the draw-down experiment in the present study since this configuration is the optimal one to estimate k_l and b simultaneously.

While the experiment consists in recording $P_0(t)$=P(0,t) from which the determination of k_l (and b) is to be performed, the solution of the above problem is clearly needed. Except under simplifying assumptions discussed above, this can not be achieved analytically. Hence, for completeness, we use a direct numerical resolution of equations (1) with the associated initial and boundary conditions, taking an extremely large value of V_1 (typically 1 m^3). The numerical method is a second order finite difference explicit scheme (Jannot *et al.*, 2007).

EXISTING INTERPRETATIVE MODELS - INVERSE PROCEDURE

Because simplified solutions are appealing for practical routine interpretation of pressure decay, we first analyze an approximate solution, made popular by Jones (1972). This solution relies on the hypothesis of a constant mass-flow rate along the sample yielding the following relationship to interpret the data record of $P_0(t)$:

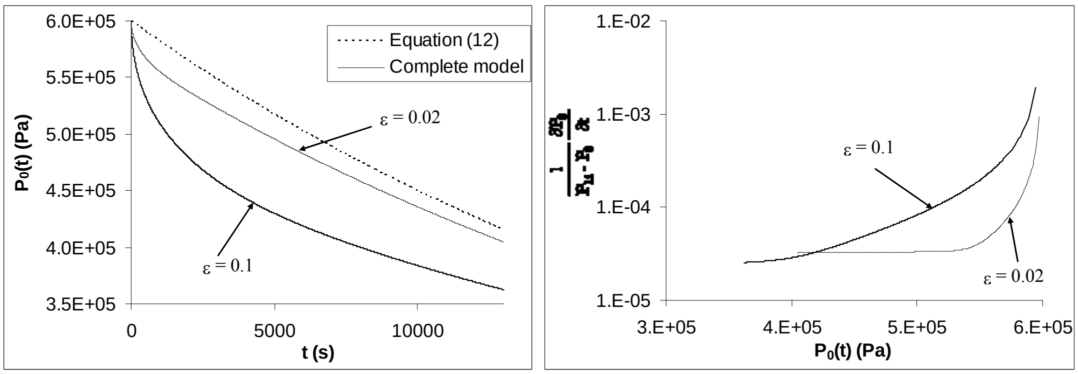
$$-\frac{\partial P_0}{\partial t} = \frac{k_l S}{2\mu e V_0}\left(P_{1i} + 2b + P_0\right)\left(P_0 - P_{1i}\right) \tag{2}$$

Very recently, exactly the same solution was proposed under the integrated form given by (Kaczmarek, 2008):

$$P_0(t) = \frac{(2b + P_{1i})\dfrac{P_{0i} - P_{1i}}{P_{0i} + 2b + P_{1i}} + P_{1i}\exp\left(\dfrac{k_l S}{\mu e V_0}(b + P_{1i})t\right)}{\exp\left(\dfrac{k_l S}{\mu e V_0}(b + P_{1i})t\right) - \dfrac{P_{0i} - P_{1i}}{P_{0i} + 2b + P_{1i}}} \tag{3}$$

The constant mass flow-rate hypothesis is equivalent to assuming that the accumulation term in the mass balance equation is zero, *i.e.* that the porosity is zero. This, of course, is a major source of error as shown in figure 1a. In this figure we have reported $P_0(t)$ obtained from the direct numerical resolution of the complete model for $k_l = 10^{-19}$ m^2, b = 1.31×10^6

Pa, D = 0.05 m, e = 0.05 m, $V_0 = 10^{-5}$ m^3, $P_{0i} = 6x10^5$ Pa, $P_{1i} = 10^5$ Pa , $\varepsilon = 0.02$ and $\varepsilon = 0.1$ along with the approximation of equation (3). Note that this signal is synthetic, without noise that would be present for a real measured one. One can clearly see in figure 1*a* that the pressure decay is not well described by this estimation and that, as expected, the discrepancy increases with ε.



a) *b)*

Figure 1. a) Direct simulations (continuous lines) of $P_0(t)$ compared to the prediction given by equation (3) (dashed line). b) $\partial P_0/\partial t/\left(P_{1i}-P_0\right)$ *versus $P_0(t)$ for the same signals..*

To further illustrate this behaviour, we have used the procedure suggested by equation (2) from which k_l and b are estimated from a linear fit performed on the plot of $\partial P_0/\partial t/\left(P_{1i}-P_0\right)$ versus $P_0(t)$. As expected, since accumulation in the pores is an important mechanism at the early stage of the pressure decay remaining over a period of time that increases with ε, a linear fit is impossible to perform on this part of the signal (see figure 1*b*). Such a fit is meaningful on the late part of the signal requiring that the experiment is carried on over a large enough period of time. This represents a major drawback since the period required for such an interpretation to hold is unknown a priori. To improve the method, an iterative correction to the relationship in equation (2) was proposed accounting for mass flow rate change along the sample (see Appendix in Jones (1972)). Nevertheless, one must keep in mind that a real measurement provides a noisy $P_0(t)$ signal. Because the whole procedure requires the derivation of the pressure decay signal, the estimation remains difficult in spite of some interesting approximations proposed also in Jones (1972) to make the technique more tractable. The overall procedure described in this reference was validated on a range of initial pressure P_{0i} close to $8x10^5$ Pa and intrinsic permeabilities larger than 10^{-19} m^2. In the general case however, the impact of the successive approximations on the final result is difficult to estimate. Along with the fact that the information contained in the early stage of the pressure decay record is not used, this makes an alternative method desirable, taking advantage of the whole signal that can be shorter than the required time for the approximated model to apply as will be shown below. This is the purpose of the inverse technique developed in this work.

Our inverse technique is a Levenberg-Marquardt algorithm and makes use of the complete pulse-decay model although results will only be presented within the framework of a draw-down experiment described above. This algorithm was developed to estimate simultaneously k_l and b assuming that ε is known *a priori*. Details on the algorithm can be found for instance in Marquardt (1963) or Gill and Murray (1978). Basically, it consists in minimizing the square of the χ function given by $\chi^2(k_l, b) = \sum_{i=1}^{N} \left(\left(P_{0m}(t_i) - P_0(t_i, k_l, b) \right) / \sigma_i \right)^2$ where N is the number of time records of the measured signal P_{0m}, P_0 is the value of the upstream pressure estimated from the model at a given pair (k_l, b) and σ_i is the standard deviation on P_{0m} at the i^{th} time-record. Convergence is achieved once $\chi^2(k_l, b)$ falls below a user-defined criterion chosen here equal to 0.05. To ensure convergence of the inverse procedure performed on the complete model, k_l and b are pre-estimated with the same inverse algorithm using the simplified model given by equation (2) or (3) on the late part of the signal. Alternatively, when the signal is not available on a sufficiently long period for such an estimation to be physically meaningful (see discussion above), guessed values are provided by the user.

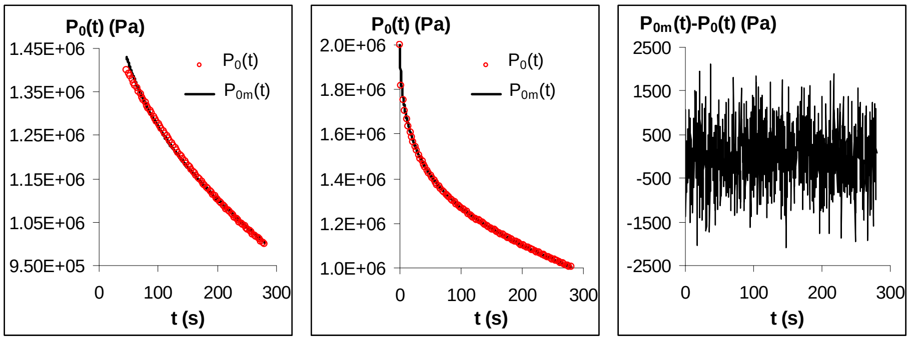
Our purpose is now to investigate the impact of errors in the different parameters (D, e, V_0, ε, etc.) on the estimation of k_l and b.

SENSITIVITY OF THE INVERSE PROCEDURE

In order to carefully analyze the impact of the different parameters, inversion is performed on synthetic signals obtained from direct numerical simulations of the complete model. However, to be more representative of a real measurement, a gaussian noise was superimposed to the synthetic signal. This noise is given by $\delta P = 0.01\, dP\, s\, P_{0i}/3$ where s is a random number of unit standard deviation and dP is the error on $P_0(t)$ (in % of the measurement). The coefficient 3 was taken so that $P_{0m}(t) \pm \delta P$ includes 99.7% of $P_0(t)$ values if they would have been measured. Here, we chose dP=0.1% and we continue to note $P_{0m}(t)$ the resulting noisy signal. For completeness, the analysis is performed on three different configurations: i) **material 1**: $k_l = 10^{-17}$ m^2 ; b = 2.49 x 10^5 Pa ; ε = 0.02, $P_{0i} = 10^6$ Pa, t_f = 500 s; ii) **material 2**: $k_l = 10^{-17}$ m^2 ; b = 2.49 x10^5 Pa ; ε = 0.1, P_{0i}=2 10^6 Pa, t_f = 280 s; iii) **material 3**: $k_l = 10^{-19}$ m^2; b = 13.08 x10^5 Pa, ε = 0.02, $P_{0\,i}$= 3 x 10^6 Pa, t_f = 13000 s. The values of the period of record, t_f, corresponds to $\left(P_{0m}(t_f) - P_{1i}\right)/\left(P_{0i} - P_{1i}\right) \approx 0.5$. In each case, we took $P_{1i} = 10^5$ Pa and μ = 1.8 10^{-5} Pa.s while the nominal values of D, e and V_0 are D = 0.05 m, e = 0.05 m and $V_0 = 10^{-5}$ m^3.

The validity of our inverse modelling is illustrated in figure 2 in the case of material 2 with N=1000. Here, we have used 200 grid points in space for both the signal generation and its inversion. In figure 2*a*, we have reported the late part of the initial signal $P_{0m}(t)$ (corresponding to $P_{0m}(t) < P_{1i} + 0.7(P_{0i} - P_{1i})$) along with the pre-estimation obtained with the inversion performed with the simplified model. Although the fit seems satisfactory, the estimated value of b = -5.2x10^5 Pa is physically unacceptable while k_l (1.4 10^{-17} m^2) is rather far from the input value. This is a clear indication that the data of $P_{0m}(t)$ is given over too short period of time, making the estimation with the simplified model (equations (2) or (3)) impossible. Conversely, when the complete model is used in the inverse procedure

(see figure 2*b*), one recovers excellent results compared to the input data since the estimated values are respectively $k_l = 1.0 \times 10^{-17}$ m^2 and $b = 2.487 \times 10^5$ Pa.
Note finally that residuals at the end of the inverse procedure are randomly distributed around zero, as a signature of the noise (see figure 2*c*). The computational time to perform the inversion is on the order of 10 min on a 1700 MHz Pentium® processor.



a) *b)* *c)*

Figure 2. Inverse procedure on material 2. a) Initial data $P_{0m}(t)$ and pre-estimation $P_0(t)$. b) Initial data $P_{0m}(t)$ and estimated signal $P_0(t)$ using the inverse technique with the complete model. c) Residuals between the initial data and the estimated signal.

In the sequel of this paper, signals were generated using 1000 grid points in space while inversion was carried out with 100 points for materials 1 and 3 and 200 points for material 2, keeping N=1000. As can be seen from the following tables, this introduces a bias in the estimation of k_l and b (from roughly 2 to 4% for k_l and 8 to 9% for b) suggesting a significant impact of spatial discretization on the procedure that will not be further detailed in this work however.

Time sampling

Here, we investigate the impact of the time sampling used to record the pressure decay $P_{0m}(t)$ on the values of k_l and b estimated with the inverse procedure.

Table 1. Influence of time sampling of $P_{0m}(t)$ on the estimated values of k_l and b.

	Number of time steps N	Estimated values		Relative error (%)	
		k_l (m^2)	b x 10^5 (Pa)	k_l	b
Material 1	100	1.03E-17	2.2669	-2.6	9.0
	200	1.03E-17	2.2515	-3.0	9.6
	500	1.03E-17	2.244	-3.0	9.9
	700	1.03E-17	2.2412	-3.0	10.0
	1000	1.03E-17	2.2471	-3.0	9.8
Material 2	100	1.02E-17	2.2888	-2.0	8.1
	200	1.02E-17	2.2653	-2.0	9.0
	500	1.02E-17	2.2734	-2.0	8.7
	700	1.02E-17	2.2623	-2.0	9.1
	1000	1.02E-17	2.2614	-2.0	9.2
Material 3	100	1.03E-19	12.083	-3.5	7.6
	200	1.04E-19	11.9823	-3.9	8.4
	500	1.04E-19	12.0219	-3.7	8.1
	700	1.04E-19	11.9853	-3.9	8.4
	1000	1.04E-19	11.9884	-3.9	8.3

This is performed by considering successively $P_{0m}(t)$ sampled with N = 100, 200, 500, 700 and 1000 for each of the three materials. Results of this analysis are summarized in table 1. Obviously, no influence of time sampling is observed on the resulting estimation of k_l and b. In other words, no precision can be gained from increasing the number of points to sample the time evolution of $P_{0m}(t)$ over a given period of time.

Error on D and e

We now analyse the sensitivity of the inverse procedure to an error δD in D and δe in e. Tests were performed with $\delta D = \delta e = 10^{-4}$ m that represents a reasonable value of the accuracy one can expect on the determination of the sample dimensions. The analysis is carried out by first generating $P_{0m}(t)$ using the nominal values of D and e and then performing the inversion procedure using D ± δD and e ± δe. Results of the tests are gathered in table 2. They show that an error on both D and e has little impact on the estimated values of k_l and b since the additional error with respect to the case where nominal values of these parameters are used is less than 1.5% for k_l and 3.3% for b. This completes the conclusion of our sensitivity analysis (Jannot *et al.*, 2007) where it was shown that the sample dimensions have almost no impact on the expected precision on k_l and b (*i.e.* that there is no optimal values of D and e to better estimate k_l and b).

Table 2. Impact of errors in D, e and V_0 on the estimated values of k_l and b.

	Input values			Estimated values		Relative error (%)		D, e (mm), V_0 (cm^3) used for inversion
	k_l (m^2)	b x10^5 (Pa)	ε	k_l (m^2)	b x10^5 (Pa)	k_l	b	
Material 1	1.00E-17	2.49	0.02	1.03E-17	2.2471	-3.0	9.8	50, 50, 10
				1.02E-17	2.3008	-1.7	7.6	50.1, 50, 10
				1.04E-17	2.1897	-4.3	12.1	49.9, 50, 10
				1.03E-17	2.2698	-2.8	8.8	50, 50.1, 10
				1.03E-17	2.2165	-3.2	11.0	50, 49.9, 10
				1.03E-17	2.2296	-3.3	10.5	50, 50, 10.1
				1.03E-17	2.2566	-2.7	9.4	50, 50, 9.99
Material 2	1.00E-17	2.49	0.10	1.02E-17	2.2614	-2.4	9.2	50, 50, 10
				1.01E-17	2.3442	-0.9	5.9	50.1, 50, 10
				1.04E-17	2.1791	-3.9	12.5	49.9, 50, 10
				1.02E-17	2.3038	-2.0	7.5	50, 50.1, 10
				1.03E-17	2.2221	-2.7	10.8	50, 49.9, 10
				1.03E-17	2.2413	-2.7	10.0	50, 50, 10.1
				1.02E-17	2.2815	-2.0	8.4	50, 50, 9.99
Material 3	1.00E-19	13.08	0.02	1.04E-19	11.9883	-3.9	8.3	50, 50, 10
				1.02E-19	12.2185	-2.5	6.6	50.1, 50, 10
				1.04E-19	11.9884	-3.9	8.3	49.9, 50, 10
				1.04E-19	12.1016	-3.6	7.5	50, 50.1, 10
				1.04E-19	11.8732	-4.2	9.2	50, 49.9, 10
				1.04E-19	11.9512	-4.2	8.6	50, 50, 10.1
				1.04E-19	12.0449	-3.5	7.9	50, 50, 9.99

Error in V_0

The impact of an error δV_0 in V_0 is now checked using the same methodology as the one mentioned above. We chose $\delta V_0 = 10^{-8}$ m^3 which seems a reasonable accuracy on a careful measurement of V_0. The signal $P_{0m}(t)$ obtained with the nominal value of V_0 is used as the input of the inverse procedure that is run using $V_0 \pm \delta V_0$ to determine k_l and b. Results on the estimated values of k_l and b along with the corresponding relative errors are reported in table 2. As for errors in the dimensions of the sample, an error in V_0 has a weak effect on the estimation of k_l and b (the additional error compared to the case where the nominal value of V_0 is used is only 0.3% for k_l and 0.7% for b). Again, this completes the sensitivity analysis reported by Jannot *et al.* (2007) indicating that the value of V_0 does not affect the expected precision on the estimation of k_l and b in a draw-down experiment.

Error in ε

In this section, we want to analyse the impact of an error in the porosity ε on k_l and b estimated with the inverse procedure. Again, the signal $P_{0m}(t)$ is simulated with the nominal value of ε (and of all the other parameters) while the inversion is performed using $\varepsilon \pm \delta\varepsilon$. The error in ε was estimated by $\delta\varepsilon = 0.2\varepsilon$ when $\varepsilon \leq 0.05$ and by $\delta\varepsilon = 0.05\varepsilon$ otherwise. Inversion results are presented in table 3. The relative errors on the estimated values of k_l and b clearly indicate that these quantities are strongly affected by the precision of ε. Consequently, it is highly recommended to provide a precise value of the porosity for a reliable estimation of k_l and b using this type of experiment and the inverse modelling presented here.

Table 3. Impact of errors in porosity ε and dead upstream volume dV on the estimated values of k_l and b.

	Input values			Estimated values		Relative error (%)		ε, dV (cm^3)
	k_l (m^2)	b x10^5 (Pa)	ε	k_l (m^2)	b x10^5 (Pa)	k_l	b	used for inversion
Material 1	1.00E-17	2.49	0.02	6.24E-18	6.3464	37.6	-154.9	0.024, 0
				1.48E-17	0.3318	-47.7	86.7	0.016, 0
				1.34E-17	0.7761	-34.2	68.8	0.02, 0.1
Material 2	1.00E-17	2.49	0.10	8.94E-18	3.3480	10.6	-34.5	0.105, 0
				1.18E-17	1.2524	-17.7	49.7	0.095, 0
				1.19E-17	0.9998	-18.9	59.8	0.1, 0.1
Material 3	1.00E-19	13.08	0.02	6.06E-20	2.8300	39.4	78.4	0.024, 0
				1.59E-19	4.0245	-58.6	69.2	0.016, 0
				1.41E-19	5.9018	-41.2	54.9	0.02, 0.1

Upstream dead volume

We end our analysis by a short investigation of the effect of the presence of a dead volume dV between the valve closing the upstream tank and the upstream (entrance) face of the sample. In practice, this volume cannot be totally eliminated and it is hence of major interest to study its impact on the estimation of k_l and b. In our numerical code, a dead volume dV can be accounted for by simply expressing the fact that, when present, the experience is equivalent to that without a dead volume where the volume of the upstream tank is V_0+dV and the initial pressure is $(P_{0i}\ V_0 + P_{1i}\ dV)/(V_0 + dV)$. As a consequence, the input signal $P_{0m}(t)$ is generated with a dead volume and the inversion is performed with the nominal value of V_0 and P_{0i} to determine k_l and b. Our test was carried out with $dV = 0.01V_0 = 10^{-7}\ m^3$‡. Results for the estimated values of k_l and b obtained on the three materials under consideration are reported in table 3 along with the corresponding relative errors. Obviously, the presence of a dead volume has a very significant impact on the determination of the intrinsic permeability and Klinkenberg coefficient, this later parameter being more affected than k_l. As a consequence, should such a dead volume be present in a draw-down experimental device, it is strongly recommended to carefully determine this volume and include it in the interpretation of the signal with an inverse procedure in order to achieve reliable estimates of k_l and b.

CONCLUSIONS

In this work, we have shown how an inverse technique can be used to estimate both the intrinsic permeability k_l and Klinkenberg coefficient b from a single unsteady-state gas flow experiment. The gain of the technique was highlighted in comparison to a classical approximated method that operates only on pressure decay signals recorded over a sufficiently long period of time. Knowing that the draw-down experiment is the optimal configuration for a better estimation of k_l and b, the analysis of the inverse procedure on this experiment indicates that:

- time sampling of the pressure decay is not a crucial parameter for a successful inversion;
- errors in the input values of the sample diameter or sample length used to carry out the present inverse modelling is of little impact on the final estimated values of k_l and b;
- an error in the input value of V_0 is of no significant influence on the final values of k_l and b extracted from the inverse procedure;
- a precise knowledge of the porosity ε of the sample is required to perform an effective estimation of k_l and b using this inverse procedure. The impact of this parameter on the final values of k_l and b is relatively strong, in particular for the latter;
- should a dead volume between the upstream face of the sample and the tank of volume V_0 be present in the experiment, it must be known precisely and provided as an input in the inverse procedure since estimated values of k_l and b are strongly affected by this dead volume.

The present technique seems a promising tool for efficient one phase-flow core analysis in particular for tight materials.

NOMENCLATURE

b	Klinkenberg coefficient	Pa	k_l	Intrinsic or liquid permeability	m^2
D	Sample diameter	m	P	Pressure at x and t	Pa
e	Sample length	m	t_f	Duration of the experiment	s
V	Volume	m^3	N	Number of time record data points	
ε	Porosity		ΔP	Pressure difference	Pa
S	Sample cross sectionnal area	m^2			

‡ As an order of magnitude indication, this value corresponds to roughly 4.2 cm of a 1/8" Swagelok® tubing.

Indices

m	Measured (here, simulated with noise)	0	High pressure reservoir
i	Initial values at time t = 0	1	Low pressure reservoir

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