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# A centered hot plate method for measurement of thermal properties of thin insulating materials

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**Abstract**
This paper presents a method dedicated to thermal conductivity measurement of thin (a few millimeters thickness) insulating and super-insulating materials. The method is based on the measurement of the temperature at the center of a heating element inserted between two samples, with the unheated surface of the samples maintained constant. A 3D model of the heat transfer in the system has been established and simulated to determine the validity conditions of a 1D model to represent the center temperature. This 1D model was then used to realize a sensitivity analysis of the center temperature to the different parameters. The conclusion is that the thermal conductivity may be estimated with a good precision for all insulating materials from a simple steady state measurement and that the thermal capacity may also be estimated from transient recording of the temperature with a precision increasing with the value of the thermal capacity of the samples. It has then been shown that a device with two samples of different thickness improves the precision of the estimation of the thermal capacity. These conclusions are validated by an experimental study on polyethylene foam and PVC samples leading to an estimation of their thermal properties very close to the values measured by other classical methods (deviation < 5%).

**Keywords:** thermal conductivity, thermal capacity, hot plate, transient method, insulating material

(Some figures in this article are in colour only in the electronic version)

## Nomenclature

| | |
|---|---|
| $a$ | thermal diffusivity (m$^2$ s$^{-1}$) |
| $b$ | sample and heat element half-width (m) |
| $c$ | specific heat (J kg$^{-1}$ K$^{-1}$) |
| $C_t$ | total thermal capacity (sample + heating element) (J K$^{-1}$) |
| $d$ | sample and heat element half-length (m) |
| $e$ | thickness (m) |
| $N$ | number of experimental points |
| $S$ | sample and heating element area (m$^2$) |
| $T$ | sample temperature (°C) |
| $T_i$ | initial temperature of the system (°C) |
| $\Delta T$ | temperature difference between the heating element and the blocks (°C) |
| $\phi_0$ | heat flux density in the heating element (W m$^{-2}$) |
| $\varphi_0$ | heat flux in the heating element (W) |
| $\Phi$ | Laplace transform of the heat flux |
| $\lambda$ | thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $\rho$ | density (kg m$^{-3}$) |
| $\sigma_X$ | standard deviation on the parameter $X$ |
| $\theta$ | Laplace transform of the temperature $T$ |

### Subscripts

| | |
|---|---|
| $b$ | isothermal blocks |
| $c$ | center |
| exp | experimental |
| $h$ | heating element |
| mod | model |

 

## 1. Introduction

The thermal conductivity of thin insulating and super-insulating materials (materials with thermal conductivity lower than air conductivity) is difficult to measure, particularly for very low density materials. Many different methods are available for thermal conductivity measurement of insulating materials; the more frequently used and the latest published are as follows.

- The guarded hot plate method [1–3]
- The hot wire method [4–7]
- The hot strip method [8, 9]
- The hot disk method [10–12], the tiny hot plate method [13] and the three-layer device method [14].

These methods present the following disadvantages.

- The guarded hot plate method needs large samples (50 × 50 cm$^2$).
- The hot wire method enables the estimation of the thermal conductivity from the slope of the curve $T(t) = f[\ln(t)]$ assuming two hypotheses: the sample is semi-infinite and the sensitivity of the temperature to the thermal capacity of the probe is negligible. This last hypothesis is verified only after a time that increases when the density of the material decreases. Even if the use of a complete model (taking into account the thermal capacity of the probe) enables us to process the temperature recording from the beginning, the validity of the first hypothesis (semi-infinite medium) may impose large sample dimensions. The conductivity estimation may be wrong if the sample is not large and thick enough or if the estimation time is too short [7].
- The hot strip and hot disk methods are based on a model, and an estimation method quite complex and the uncertainties in the probe dimensions may lead to an uncertainty in the thermal conductivity estimation.
- The tiny hot plate and three-layer device methods are based on the processing of mean temperatures that impose taking into account the convective heat transfer on the lateral faces of the sample. Moreover, in the case of super-insulating materials, the conductive heat transfer in the air surrounding the sample is not negligible compared to the heat transfer inside the sample and must be taken into account. The heat transfer in the air is quite difficult to model since the boundary conditions in the air are not well known.
- The hot wire, hot strip and hot disk methods cannot lead to an estimation of the thermal conductivity in one direction from a unique experience for an anisotropic material.

The aim of this work was to propose a method.

- Easy to use and suited to relatively small samples, particularly to low thickness samples.
- Suited to low-conductivity measurement.
- Using an estimation method based on a simplified model whose validity may be verified *a posteriori* by a more complex model.

## 2. Materials and methods

The different elements that make up the experimental device represented in figure 1 are as follows.

- A heating element made of a plane resistance inserted between two insulating polymide films. Its thickness is $e_h = 0.22$ mm and a type K thermocouple (wire diameter of 0.03 mm) is fixed at the center. The heating element is inserted between two samples with a thickness $e$. The heating element and the samples have the same cross-section area $S$.
- Two isothermal aluminum blocks with a thickness 40 mm and the same cross-section $S$ as the samples.
- A tightening device enabling pressure control and the measurement of the thickness of the device inserted between the aluminum blocks.

A flux step is sent in the heating element and the temperatures $T_{c_{\exp}}(t)$ at the center of the heating element and $T_{b_{\exp}}(t)$ of the aluminum blocks are recorded. The processing of the recording of $T_{c_{\exp}}(t)$ and $T_{b_{\exp}}(t)$ is realized by supposing that the heat transfer at the center of the heating element is 1D. A 3D model will enable us to verify this hypothesis. A simplified 1D model is then used to estimate the thermal characteristics: a stationary model is sufficient to estimate the thermal conductivity $\lambda$, and a transient model is used to estimate the thermal capacity $\rho c$.

The following hypotheses have been considered.

- The system is at a uniform temperature $T_i$ (equal to the ambient air temperature) at initial time.
- The sample is opaque.
- Thermal contact resistances and thermal resistance of the heating element are negligible compared to the sample thermal resistance.

*3D model*

The system will be first modeled with the hypothesis that the thermal capacity of the aluminum blocks is large enough so that the temperature $T_{b_{\exp}}$ remains constant during the experiment (cf figure 2). If $T(x, y, z, t)$ is the temperature of the sample, the heat transfer equation is

$$\frac{\partial^2 T(x,y,z,t)}{\partial x^2} + \frac{\partial^2 T(x,y,z,t)}{\partial y^2} + \frac{\partial^2 T(x,y,t)}{\partial z^2} = \frac{1}{a}\frac{\partial T(x,y,z,t)}{\partial t} \quad (1)$$

where $a$ is the thermal diffusivity (m$^2$ s$^{-1}$) of the sample.

The initial condition is

$$T(x,y,z,0) = T_i. \quad (2)$$

The boundary conditions are

$$\frac{\partial T(0,y,z,t)}{\partial x} = 0 \quad (3)$$

$$\frac{\partial T(x,0,z,t)}{\partial y} = 0 \quad (4)$$

$$-\lambda \frac{\partial T(b,y,z,t)}{\partial x} = h[T(b,y,z,t) - T_i] \quad (5)$$

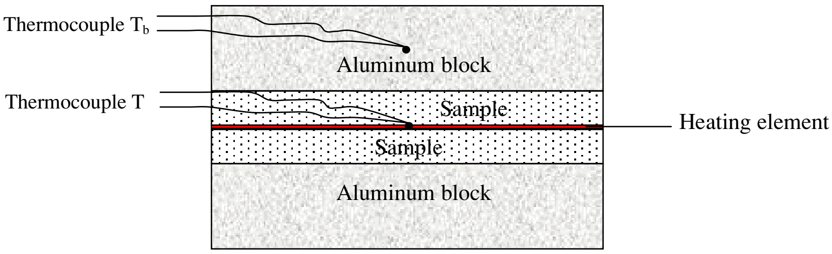


**Figure 1.** Schema of the experimental device.

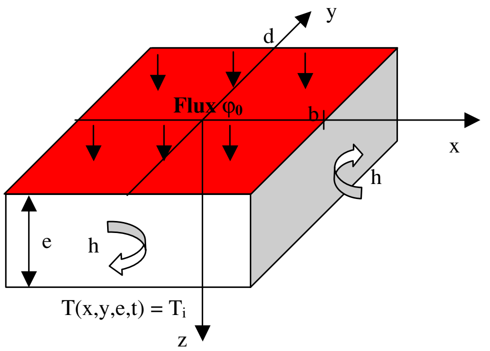


**Figure 2.** Schema of the modeled system.

$$-\lambda \frac{\partial T(x,d,z,t)}{\partial y} = h[T(x,d,z,t) - T_i] \quad (6)$$

$$T(x,y,e,t) = T_i. \quad (7)$$

Since the thermal contact resistances and thermal resistance of the heating element are supposed negligible, the temperature of the heating element $T_h(x, y, t)$ is equal to $T(x, y, 0, t)$ and

$$\frac{\phi_0}{2} = \frac{1}{2}\rho_h e_h c_h \frac{\partial T(x,y,0,t)}{\partial t} - \lambda \frac{\partial T(x,y,0,t)}{\partial z} \quad (8)$$

where $\lambda$ is the sample thermal conductivity (W m$^{-1}$ K$^{-1}$); $e$, $2b$ and $2d$ are respectively the thickness, the width and the length (m) of the samples; $\rho_h$, $c_h$ and $e_h$ are respectively the density (kg m$^{-3}$), the specific heat (J kg$^{-1}$ K$^{-1}$) and the thickness (m) of the heating element; $h$ is the convective heat transfer coefficient (W m$^{-2}$ K$^{-1}$) and $\phi_0$ is the heat flux density produced in the heating element (W m$^{-2}$).

Setting

$$\Delta T(x,y,z,t) = T(x,y,z,t) - T_i \quad \text{and}$$
$$\mathcal{L}[\Delta T(x,y,z,t)] = \theta(x,y,z,p), \quad (9)$$

where $\mathcal{L}$ is the Laplace operator, the Laplace transform of relation (1) leads to

$$\frac{\partial^2\theta(x,y,z,p)}{\partial x^2} + \frac{\partial^2\theta(x,y,z,p)}{\partial y^2} + \frac{\partial^2\theta(x,y,z,p)}{\partial z^2} = \frac{p}{a}\theta(x,y,p). \quad (10)$$

Using the separation of variables method, the Laplace transform of the temperature may be written as

$$\theta(x,y,z,p) = X(x,p)Y(y,p)Z(z,p). \quad (11)$$

The resolution of the system of equations (1) to (11) leads to

$$\theta(x,y,z,p) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \times \frac{\frac{\Phi_0(p)}{2}\frac{\sin(\alpha_p b)}{\alpha_p}\frac{\sin(\delta_q d)}{\delta_q}\cos(\alpha_p x)\cos(\delta_q y)\sinh[\gamma_{pq}(e-z)]}{G_{pq}\left[\frac{\sin(2\alpha_p b)}{4\alpha_p} + \frac{b}{2}\right]\left[\frac{\sin(2\delta_q d)}{4\delta_q} + \frac{d}{2}\right]} \quad (12)$$

where

$$G_{pq} = \lambda\gamma_{pq}\sinh(\gamma_{pq}e) + \frac{\rho_h c_h e_h}{2} p\cosh(\gamma_{pq}e). \quad (13)$$

$\alpha_n$ are the solutions of the equation $\alpha b \tan(\alpha b) = H_x$, with

$$H_x = \frac{hb}{\lambda}. \quad (14)$$

$\delta_n$ are the solutions of the equation $\delta d \tan(\delta d) = H_y$, with

$$H_y = \frac{hd}{\lambda}. \quad (15)$$

The values of $\gamma$ are then given by

$$\gamma_{pq}^2 = \frac{p}{a} + \alpha_p^2 + \delta_q^2. \quad (16)$$

$\Phi_0(p)$ is the Laplace transform of the heat flux in the heating element; in the case of a flux step its expression is

$$\Phi_0(p) = \frac{\phi_0}{p}. \quad (17)$$

One can deduce the expression of the Laplace transform of the temperature at the center of the heated face of the sample:

$$\theta(0,0,0,p) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \frac{\frac{\Phi_0(p)}{2}\frac{\sin(\alpha_p b)}{\alpha_p}\frac{\sin(\delta_q d)}{\delta_q}\cosh(\gamma_{pq}e)}{G_{pq}\left[\frac{\sin(2\alpha_p b)}{4\alpha_p} + \frac{b}{2}\right]\left[\frac{\sin(2\delta_q d)}{4\delta_q} + \frac{d}{2}\right]}. \quad (18)$$

*1D model*

A simplified model may be established by considering the supplementary hypothesis that the heat transfer remains 1D at the center of the system during the experiment. With this hypothesis, the center temperature depends only on $z$ and $t$ and will be denoted by $T_c(z, t)$ and its Laplace transform will be denoted by $\theta_c(z, p)$.

The temperature in the aluminum blocks is no longer supposed time independent but is supposed uniform. This last hypothesis is validated if the Biot number $Bi = \frac{hb}{\lambda}$ is lower than 0.1. Considering $h = 10$ W m$^{-2}$ K$^{-1}$, thermal conductivity of the block $\lambda_b = 200$ W m$^{-1}$ K$^{-1}$ and sample dimensions $b = d = 100$ mm lead to $Bi = 0.005$ so that the temperature of the aluminum blocks may be considered as uniform.

Since the thermal contact resistance has been neglected, the temperature of the isothermal blocks $T_b(t)$ is equal to $T_c(e, t)$.

Thus, the quadrupolar equation may be written [15] as

$$\begin{bmatrix} \theta_c(0,p) \\ \Phi_c(0,p) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_h p & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_c(e,p) \\ \Phi_c(e,p) \end{bmatrix} \quad (19)$$

where $A = D = \cosh\left(\sqrt{\frac{p}{a}}e\right)$; $B = \frac{\sinh\left(\sqrt{\frac{p}{a}}e\right)}{\lambda S\sqrt{\frac{p}{a}}}$; $C = \lambda S\sqrt{\frac{p}{a}}\sinh\left(\sqrt{\frac{p}{a}}e\right)$; $S$ is the surface area of the sample and of the heating element; $C_h = \frac{1}{2}\rho_h c_h e_h S$ where $\rho_h$, $c_h$ and $e_h$ are respectively the density (kg m$^{-3}$), the specific heat (J kg$^{-1}$ K$^{-1}$) and the thickness (m) of the heating element.

Furthermore, the heat flux at the center of the unheated face of the sample may be calculated as

$$\varphi_c(e,t) = C_b \frac{\mathrm{d}T_c(e,t)}{\mathrm{d}t} + h S_b T_c(e,t) \quad (20)$$

where $C_b = \rho_b c_b e_b S$ with $\rho_b$, $c_b$ and $e_b$ respectively the density (kg m$^{-3}$), the specific heat (J kg$^{-1}$ K$^{-1}$) and the thickness (m) of the isothermal blocks and $S_b$ the total exchange surface area between the ambient air and the isothermal block.

Its Laplace transform is

$$\Phi_c(e,p) = (C_b p + h S_b)\theta_c(e,p). \quad (21)$$

It may be deduced that

$$\begin{bmatrix} \theta_c(0,p) \\ \Phi_c(0,p) \end{bmatrix} = \begin{bmatrix} A & B \\ AC_h p + C & BC_h p + D \end{bmatrix} \begin{bmatrix} \theta_c(e,p) \\ (C_b p + h S_b)\theta_c(e,p) \end{bmatrix} \quad (22)$$

leading to

$$\theta_c(0,p) = \frac{A + B(C_b p + h S_b)}{AC_h p + C + (BC_h p + D)(C_b p + h S_b)} \Phi_c(0,p) \quad (23)$$

and

$$\theta_c(e,p) = \frac{1}{AC_h p + C + (BC_h p + D)(C_b p + h S_b)} \Phi_c(0,p). \quad (24)$$

*Simplified 1D model*

A simpler 1D model may also be written with the hypothesis that the temperature of the isothermal blocks remains constant, within this hypothesis one can write

$$\begin{bmatrix} \theta_c(0,p) \\ \Phi_c(0,p) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_h p & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 \\ \Phi_c(e,p) \end{bmatrix}. \quad (25)$$

This equation leads to

$$\theta_c(0,p) = \Phi_c(0,p) \frac{B}{BC_h p + D}. \quad (26)$$

**Table 1.** Characteristics of the materials considered in the sensitivity analysis.

| | $\lambda$ (W m$^{-1}$ K$^{-1}$) | $a$ (m$^2$ s$^{-1}$) | $\rho c$ (J m$^{-3}$ K$^{-1}$) |
|---|---|---|---|
| PVC | 0.184 | $1.21 \times 10^{-7}$ | $1.52 \times 10^{6}$ |
| Low-density insulator | 0.040 | $5.00 \times 10^{-7}$ | $8.00 \times 10^{4}$ |
| Super-insulator | 0.015 | $1.30 \times 10^{-7}$ | $1.15 \times 10^{5}$ |

For 'long times'

$$T_c(0, t \to \infty) = \frac{\phi_0}{\frac{\lambda}{e}}. \quad (27)$$

Actually, the temperature $T_c(e, t)$ increases after a certain time (corresponding to $p \to 0$). Figure 3 represents $T_c(0, t)$, $T_c(e, t)$ and $\Delta T_c(t) = T_c(0, t) - T_c(e, t)$ calculated with relations (23) and (24) and $T_c(0, t)$ calculated with relation (26). Calculations have been realized for two different insulating materials whose properties are given in table 1, considering aluminum blocks with a thickness $e_b = 40$ mm and a convection heat transfer coefficient $h = 10$ W m$^{-2}$ K$^{-2}$. Figure 3 shows that the difference between the temperature $T_c(0, t)$ calculated with relation (26) and the temperature difference $\Delta T_c(t)$ is negligible.

The principle of the method is thus to estimate the values of the parameters $\lambda$ and eventually $\rho c$ and $\rho_h c_h$ which minimize the sum of the quadratic errors $\Psi = \sum_{i=1}^{N} [\Delta T_{c_{\exp}}(t_i) - T_{c_{\bmod}}(t_i)]^2$ between the experimental curve $\Delta T_{c_{\exp}}(t) = T_{c_{\exp}}(0, t) - T_{c_{\exp}}(e, t)$ and the theoretical curve $T_{c_{\bmod}}(t) = T_c(0, t)$ calculated with relation (26) supposing that the heat transfer remains 1D at the center of the heating element. The validity of this hypothesis may be examined *a posteriori* using the 3D model.

For the 3D model, a number of 50 terms is enough to reach the convergence for the calculation of the Laplace transform of the temperature at the center of the heating element with relation (12). The roots of equations (14) and (15) are calculated numerically. The inverse Laplace transform is realized by use of the De Hoog algorithm [16].

The minimization of the sum $\psi$ is realized by use of the Levenberg–Marquardt algorithm.

*Discussion of hypotheses*

The thermal contact resistance between the sample and the heating element and between the sample and the isothermal block may be considered lower than $2 \times 10^{-4}$ K W$^{-1}$ m$^{-2}$ [13].

The thermal resistance of the sample is $\frac{e}{\lambda}$. Thus, the thermal contact resistances may be neglected (less than 1% error) compared to the sample thermal resistance if

$$\frac{e}{\lambda} > 0.02 \text{ K W}^{-1}\text{m}^{-2}. \quad (28)$$

The 3D model has then been used to estimate the limits of validity of the 1D model considering an upper value $h = 10$ W m$^{-2}$ K$^{-1}$ for the convective lateral heat transfer coefficient. For a given section area of the heating element, we have calculated the sample thickness that leads to an error of

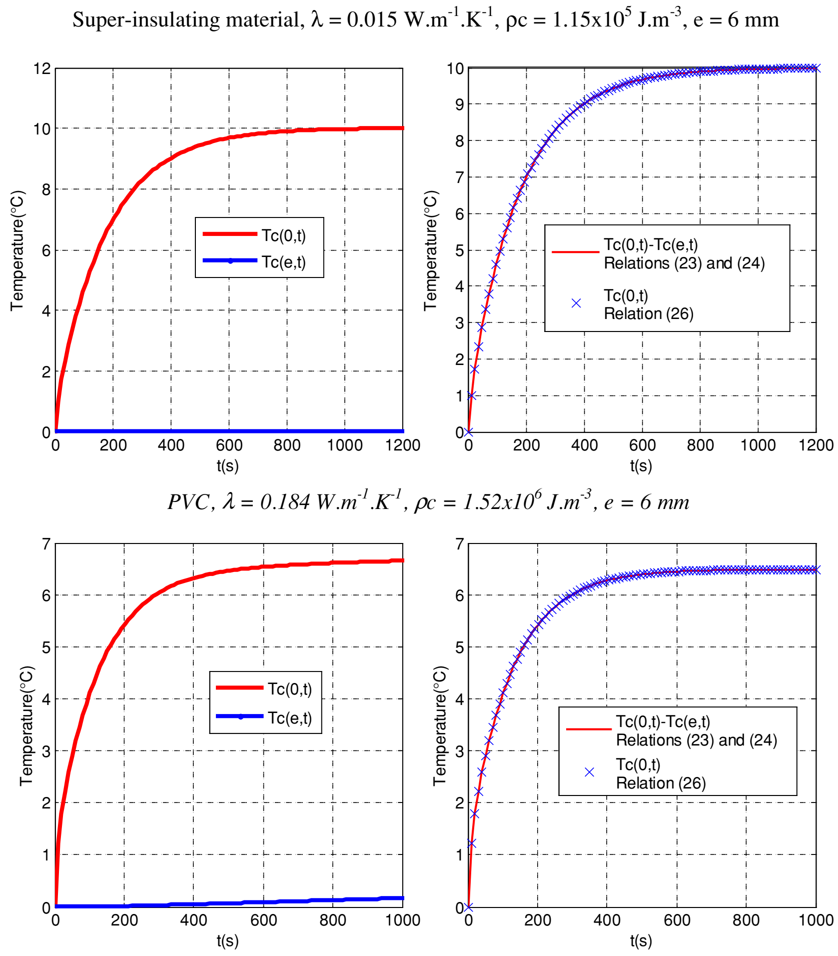


**Figure 3.** Simulations of $T_c(0, t)$ for a super-insulating material and for PVC.

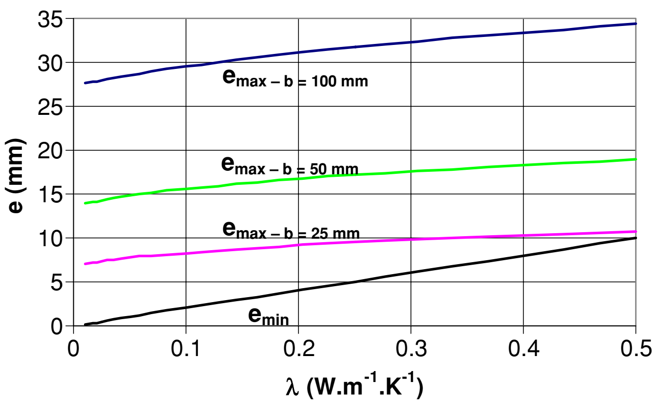


**Figure 4.** Limits of the sample thickness value to satisfy the hypotheses of the simplified 1D model (square heating element with side half-length $b$).

1% when estimating the thermal conductivity $\lambda$ with relation (27). The section of the heating element was supposed to be a square with a side half-length $b$.

In figure 4 the following are represented (as a function of the thermal conductivity of the sample).

- The minimum sample thickness required for satisfying the hypothesis that the thermal contact resistance and the heating element thermal resistance are negligible compared to the sample thermal resistance.
- The maximum sample thickness required for satisfying the hypothesis that the heat transfer remains 1D at the center of the heating element. Calculations have been realized for three different values of $b$.

Figure 4 enables us to define easily whether a couple of values $(e, \lambda)$ satisfy the hypothesis of the simplified 1D model.

## 3. Sensitivity analysis

The sensitivity analysis is based on the interpretation of the reduced sensitivity of $T(t)$ to $X$: $X\frac{\partial T}{\partial X}(t)$ providing information on the influence of the parameter $X$ on the temperature $T$ according to Beck [17].

The sensitivity analysis has been realized for three opaque insulating materials with a thickness $e = 6$ mm whose properties are reported in table 1. We considered a heating element with a thickness $e_h = 0.25$ mm and a thermal capacity $\rho_h c_h = 1.5 \times 10^6$ J m$^{-3}$ K$^{-1}$. With the hypothesis that the heat transfer is 1D at the center of the heating element, the reduced sensitivities of the center temperature $T_c(0, t)$ to the different parameters have been calculated. All the results are presented in figure 5.

One can see that

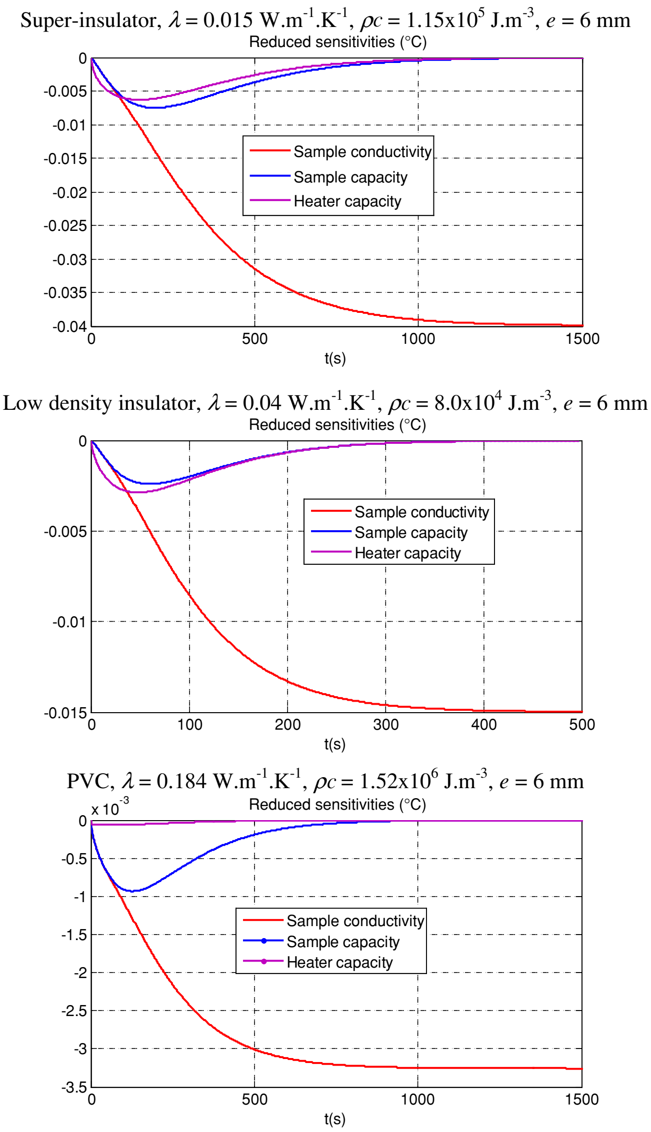


**Figure 5.** Reduced sensitivities of $T_c(0, t)$ for three insulating materials.

- the sensitivity to the thermal conductivity is high and uncorrelated to the other sensitivities,
- the sensitivity to $\rho c$ is rather proportional to the value of $\rho c$ and
- the sensitivities to $\rho_h c_h$ and to $\rho c$ are uncorrelated only for short times and will be difficult to estimate separately especially for low-density materials for which the sensitivity to $\rho_h c_h$ is superior to the sensitivity to $\rho c$.

To decorrelate the sensitivities to $\rho_h c_h$ and $\rho c$, we imagine an experiment with two samples of the same material having two different thicknesses. In this case, relation (26) is modified as follows:

$$\theta_c(0, p) = \frac{\Phi_c(0, p)}{\frac{B_1}{2B_1 C_h p + D_1} + \frac{B_2}{D_2}} \quad (29)$$

where $B_1 = \frac{\sinh\left(\sqrt{\frac{p}{a}}e\right)}{\lambda\sqrt{\frac{p}{a}}}$; $D_1 = \cosh\left(\sqrt{\frac{p}{a}}e\right)$; $B_2 = \frac{\sinh\left(\sqrt{\frac{p}{a}}2e\right)}{\lambda\sqrt{\frac{p}{a}}}$; $D_2 = \cosh\left(\sqrt{\frac{p}{a}}2e\right)$.

As an example, figure 6 represents the ratio of the reduced sensitivities to $\rho c$ and $\rho_h c_h$ calculated with relations (27) and (29) for the low-density insulating material whose properties are given in table 1.

(a) With two samples of the same thickness $e = 3$ mm.
(b) With two samples of the same thickness $e = 6$ mm.
(c) With one sample of thickness $e_1 = 3$ mm and the other sample of thickness $e_2 = 6$ mm.

It can be seen in figure 6 that the device with two samples of different thicknesses leads to a better decorrelation of the parameters $\rho_h c_h$ and $\rho c$ than the device with two samples having the same thickness. This asymmetric device must be preferred when it is possible after having verified that the temperature $T_c(e, t)$ remains constant during the experiment since it has shown that the sensitivities to the parameters $\rho_h c_h$ and $\rho c$ are correlated in a symmetrical device and that their separate estimation is not precise.

The use of the 3D model enables the verification of the similarity between the temperatures at the center of the heating element calculated with the 1D and the 3D models for the materials and the experiment duration considered in this analysis.

This sensitivity analysis predicts that the thermal conductivity λ may be estimated precisely with this method but that it will be difficult to estimate separately the thermal capacities $\rho c$ of the sample and $\rho_h c_h$ of the heating element especially for low-density materials. These conclusions will be confirmed by the experimental study.

## 4. Experimental results and discussion

Measurements have been realized using a heating element MINCO HK 5489 made of a plane resistance inserted between two insulating polymide films, with a heated surface $100 \pm 1$ mm $\times$ $100 \pm 1$ mm and a thickness $0.22 \pm 0.01$ mm.

The uncertainty in the heating element area is thus around 2%. One must add the uncertainty in the sample thickness estimated to 1% and in the heat flux produced in the heating element, estimated to 0.5%. The sum of these uncertainties leads to a global uncertainty of 3.5% to which must be added the estimation error due to the noise measurement on $\Delta T$ and the errors due to the phenomena that have not been taken into account in the model.

Measurements have been realized on four different sample couples:

- E1: polyethylene foam ($\rho = 40$ kg m$^{-3}$), two samples with a thickness 3 mm
- E2: polyethylene foam, a sample with a thickness 3 mm and a sample with a thickness 6 mm
- E3: polyethylene foam, two samples with a thickness 6 mm
- E4: PVC, two samples with a thickness 5.89 mm

The thermal conductivity and the thermal capacity of the polyethylene foam measured with the three-layer method are respectively $\lambda = 0.042$ W m$^{-1}$ K$^{-1}$ and $\rho c = 9.50 \times 10^4$ J m$^{-3}$ K$^{-1}$. The thermal diffusivity of the PVC measured by the flash method is $a = 1.21 \times 10^{-7}$ m$^2$ s$^{-1}$ and its thermal

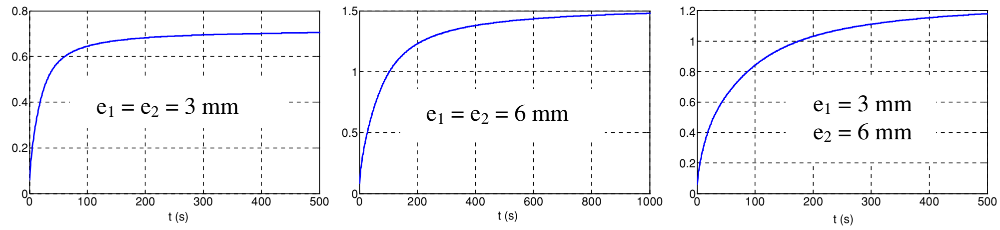


**Figure 6.** Ratio of the reduced sensitivities of $T_c(0, t)$ to $\rho c$ and to $\rho_h c_h$ calculated for different sample thicknesses of a low-density insulator.

**Table 2.** Estimation results.

| | | Relation (26) or (29) | | | | Relation (27) |
|---|---|---|---|---|---|---|
| Test Nr | Sample | $\lambda$ (W m$^{-1}$ K$^{-1}$) | $\rho c$ (J m$^{-3}$ K$^{-1}$) | $\rho_h c_h$ (J m$^{-3}$ K$^{-1}$) | $Ct$ (J K$^{-1}$) | $\lambda$ (W m$^{-1}$ K$^{-1}$) |
| 1 | E1: polyethylene foam ($e_1 = e_2 = 3$ mm) | 0.0411 | $0.603 \times 10^5$ | $1.295 \times 10^6$ | 3.87 | 0.0411 |
| 2 | | 0.0406 | $0.829 \times 10^5$ | $1.183 \times 10^6$ | 4.43 | 0.0406 |
| 3 | | 0.0406 | $1.152 \times 10^5$ | $0.980 \times 10^6$ | 5.16 | 0.0406 |
| Mean | | 0.0408 | $0.861 \times 10^5$ | $1.153 \times 10^6$ | 4.49 | 0.0408 |
| Standard deviation | | 0.71% | 32.0% | 13.9% | 14.3% | 0.71% |
| 4 | E2: polyethylene foam ($e_1 = 3$ mm; $e_2 = 6$ mm) | 0.0412 | $0.843 \times 10^5$ | $1.161 \times 10^6$ | 5.79 | 0.0412 |
| 5 | | 0.0408 | $0.856 \times 10^5$ | $1.166 \times 10^6$ | 5.86 | 0.0408 |
| 6 | | 0.0416 | $0.777 \times 10^5$ | $1.278 \times 10^6$ | 5.65 | 0.0416 |
| Mean | | 0.0412 | $0.825 \times 10^5$ | $1.202 \times 10^6$ | 5.76 | 0.0412 |
| Standard deviation | | 0.97% | 5.1% | 5.5% | 0.93% | 0.97% |
| 7 | E3: polyethylene foam ($e_1 = e_2 = 6$ mm) | 0.0402 | $0.871 \times 10^5$ | $1.201 \times 10^6$ | 7.38 | 0.0403 |
| 8 | | 0.0401 | $0.868 \times 10^5$ | $1.187 \times 10^6$ | 7.34 | 0.0404 |
| 9 | | 0.0399 | $0.761 \times 10^5$ | $1.292 \times 10^6$ | 6.81 | 0.0401 |
| Mean | | 0.0401 | $0.833 \times 10^5$ | $1.227 \times 10^6$ | 7.14 | 0.0403 |
| Standard deviation | | 0.39% | 7.6% | 4.6% | 4.5% | 0.41% |
| 10 | E4: PVC ($e_1 = e_2 = 5.89$ mm) | 0.176 | $1.515 \times 10^6$ | 0[a] | 90.8 | 0.176 |
| 11 | | 0.178 | $1.536 \times 10^6$ | | 92.2 | 0.178 |
| 12 | | 0.174 | $1.577 \times 10^6$ | | 94.6 | 0.174 |
| Mean | | 0.176 | $1.543 \times 10^6$ | | 92.6 | 0.176 |
| Standard deviation | | 1.14% | 2.03% | | 2.03% | 1.14% |

[a] In case of PVC, a too low sensitivity to $\rho_h c_h$ prevents the estimation of $\rho_h c_h$ (fixed to null value).

conductivity measured by the tiny hot plate method is $\lambda = 0.184$ W m$^{-1}$ K$^{-1}$, and its thermal capacity is thus $\rho c = 1.52 \times 10^6$ J m$^{-3}$ K$^{-1}$.

Three measures have been realized for each sample couples. The parameter values $\lambda$, $\rho c$ and $\rho_h c_h$ have been estimated by minimization of the sum of the quadratic errors between the experimental curve $\Delta T_{c_{exp}}(t) = T_{c_{exp}}(0, t) - T_{c_{exp}}(e, t)$ and the theoretical curve $T_c(0, t)$ calculated with relation (26). Figure 7 represents an example of an experimental curve and of the curve simulated with the estimated parameters; the residues defined as the difference between the experimental points and the model are also represented in the same figure. Table 2 presents a review of the experimental results.

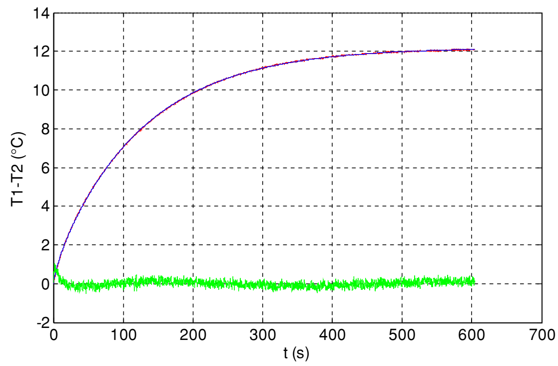


**Figure 7.** Example of experimental and simulated curves $T_c(0, t)$ and estimation residues $\times 10$ (—).

- The estimated values of the thermal conductivity $\lambda$ of the two materials are in good agreement with those measured with other methods (deviation less than 5%).
- In all cases, the value of the thermal conductivity $\lambda$ calculated with relation (26) in the semi-permanent regime is the same as the value estimated (both with the value of $\rho_h c_h$ and $\rho c$) from the transient regime.
- The reproducibility of the method is very good for the estimation of $\lambda$ with a standard deviation around 1% between the results of the three measurements.

- The thermal capacity $\rho c$ of the PVC measured by this method is very close (deviation < 1%) to the value estimated by the hot plate and the tiny hot plate methods. The deviation of $\rho c$ for the polyethylene foam is greater (around 13%).
- The estimations of $\rho c$ are more dispersed particularly for the lower density samples and strongly correlated to the values of $\rho_h c_h$. This is illustrated by the values of $\rho c$, $\rho_h c_h$ and of the global thermal capacity $C_t = S(e_h \rho_h c_h + e\rho c)$ of the ensemble heating element + sample presented in table 1. The dispersion of the values of the global thermal capacity $C_t$ is lower than the dispersion of the values of $\rho c$ and $\rho_h c_h$. The value of $\rho_h c_h$ could be previously determined by a measurement realized with a low-density material with a well-known thermal capacity $\rho c$. This value of $\rho_h c_h$ may then be considered as data leading to an estimation of only $\lambda$ and $\rho c$ to reduce the dispersion on the estimated values of $\rho c$.
- When comparing in table 1, the results obtained with the polyethylene foam for the measurements realized with the samples E2 (two samples of different thickness: $e_1 =$ 3 mm and $e_2 = 6$ mm) and E1 (two samples of the same thickness: $e = 3$ mm), it can be noticed that the device with two different thickness (E2) leads to lower standard deviation on the estimated values of $\rho c$ and $\rho_h c_h$ than the symmetrical device: 32.0% and 13.9% for E1 compared with 5.1% and 5.5% for E2. This result is in agreement with the sensitivity analysis.
- The standard deviations obtained with the asymmetrical device E2 are close to those obtained with the symmetrical device E3 (two samples of the same thickness: $e =$ 6 mm) for the values of $\rho_h c_h$: 5.5% and 4.6%, and lower for the values of $\rho c$: 5.1% and 7.6%. The standard deviation of the estimated values of the global thermal capacity *Ct* (including the heating element and the sample thermal capacity) is clearly lower in the asymmetrical device E2 than in the symmetrical device E3: 0.93% and 4.5%. The conclusion is that if $\rho_h c_h$ is known, the value of $\rho c$ may be determined more precisely with the asymmetrical device.

## 5. Conclusion

The centered hot plate method described in this paper enables the estimation of the thermal conductivity of thin insulating and super-insulating materials with quite a simple device. The thermal conductivity may be estimated with good precision using a simple relation established for a steady state measurement: the deviation of the estimated values from the values measured with other classical devices is less than 5%. The measurement of a local temperature at the center of the heating element avoids us using a hot guard or taking into account the lateral convective losses.

The validity of the hypotheses enabling the use of this simple relation may be verified *a posteriori* with the 3D model that we have developed.

The processing of the temperature transient recording also enables the estimation of the thermal capacity $\rho c$ of the tested material. The precision of this estimation will be better if the total thermal capacity $\rho ce$ of the sample is high. In all cases, this estimation will be better if an insulating material with known thermal properties is used to estimate the effective heating area $S$ of the heating element and its thermal capacity $\rho_h c_h$.

It has also been shown that an asymmetrical device with two samples of different thickness leads to a better estimation of the thermal capacity $\rho c$ of the sample especially if the thermal capacity $\rho_h c_h$ of the heating element is known.

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