Laboratoire d'Energétique et de Mécanique Théorique et Appliquée

19th
European Conference on Thermophysical Properties
August 28th – September 1st 2011
Thessaloniki, Greece

# A thermal conductivity measurement method designed for wet porous materials applied to a wood fibre based thermal insulator

Vincent FELIX
Yves JANNOT
Alain DEGIOVANNI
Christian MOYNE

Nancy-Université INPL

# Plan of the presentation

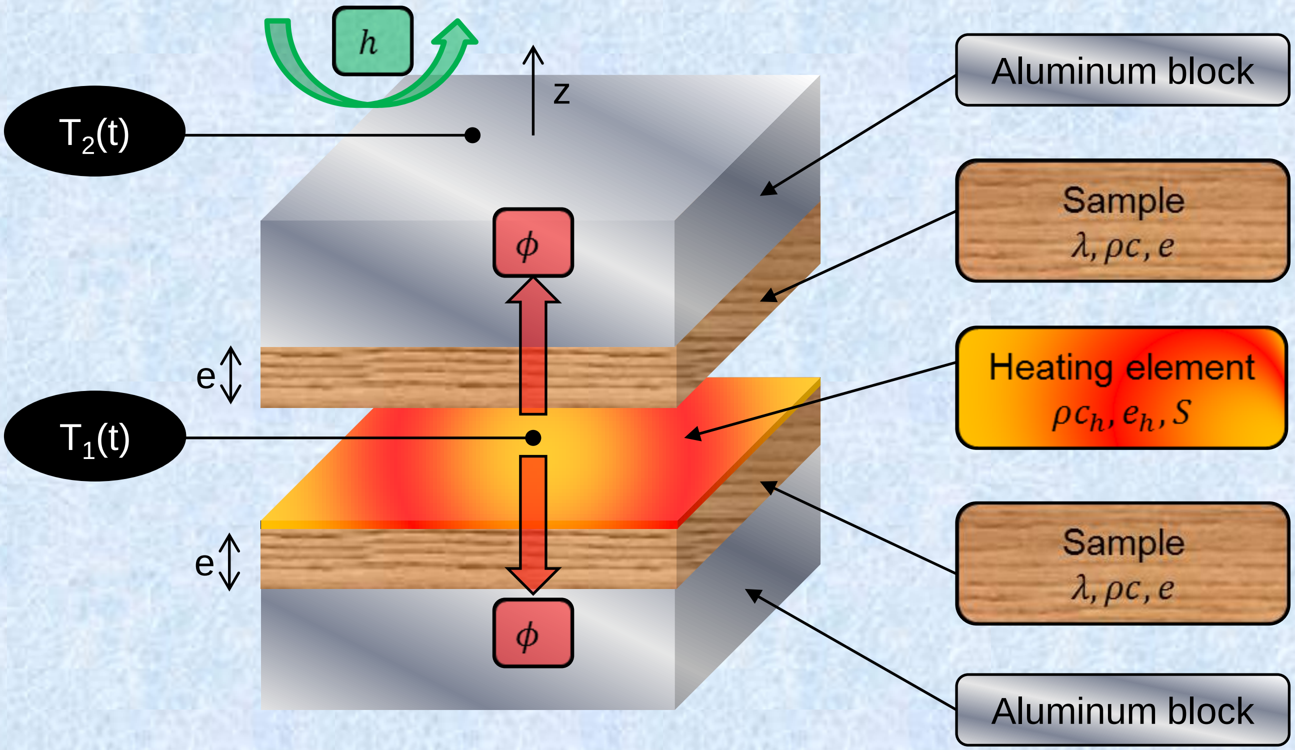
I. **Introduction**
II. **The hot plate method**
   1. Principle
   2. Modelling
III. **Estimation with a pure thermal model**
   1. Case of a dry sample
   2. Case of a wet sample
   3. Hypothesis
IV. **A coupled heat and mass transfer model**
   1. Modelling
   2. Experimental precautions
   3. Experimental results
V. **Conclusion**

# I. Introduction

- Wood fiber based materials
  - Increasingly used in buildings thermal insulation
  - Presented as ecological materials
  - Hygroscopic materials : their properties depend on ambient air humidity
- Thermal characterization
  - Need to know their thermal properties in real use conditions
  - Thermal conductivity has to be measured as a function of the water content
- How does the water modify heat transfer in a sample during a thermal measurement ?
  - Example with a hot plate method on a dry sample and a wet sample
  - Mass transfers can be taken into account in the hot plate method modelling

# II. The hot plate method

## 1. Principle




- Uniform and constant temperature $T_\infty$ for $t < 0$
- At $t = 0$, a direct voltage step is applied to the heating element
- Heat transfer at the centre of the system is supposed to be 1D along $\vec{z}$
- The system is symmetrical: $\phi = {U^2}/{2SR_e}$
- $T_1(t)$ and $T_2(t)$ are recorded
- The aluminum blocks are isothermal
- Contact resistances are negligible related to the sample's resistance

# II. The hot plate method

## 2. Modelling

Considering the following Laplace transforms:

$$\theta_1(p) = \mathcal{L}[T_1(t) - T_\infty] \qquad \Phi(0,p) = \mathcal{L}[\phi(0,t)]$$

Using the thermal quadrupoles method and assuming the hypothesis that the temperature of the isothermal block remains constant lead to simplified 1D model:

$$\begin{bmatrix} \theta_1(p) \\ \Phi(0,p) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_h & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 \\ \Phi(e,p) \end{bmatrix}$$

$$\boxed{\theta_1(p) = \frac{B}{B C_h p + D} \Phi(0,p)}$$

→ The De Hoog algorithm can be used to come back into the time dependent domain

With:

$$B = \frac{\sinh\left(\sqrt{\frac{\rho c p}{\lambda}} e\right)}{\lambda S \sqrt{\frac{\rho c p}{\lambda}}} \qquad D = \cosh\left(\sqrt{\frac{\rho c p}{\lambda}} e\right) \qquad C_h = \frac{1}{2} \rho c_h e_h S$$

- Steady state estimation:

$$\lambda = \frac{e\phi}{T_1(t \to \infty) - T_\infty}$$

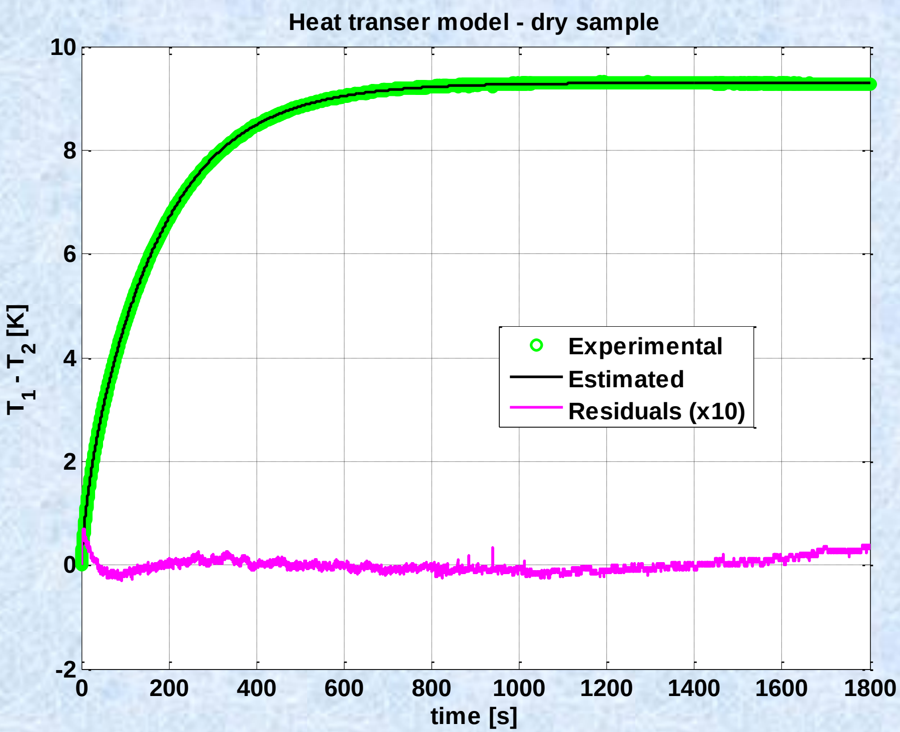
- Transient estimation:

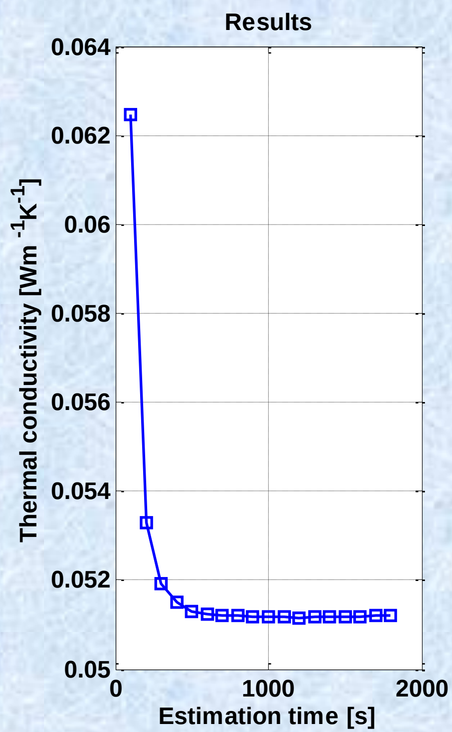
$\lambda, \rho c, \rho c_h$ estimated using a least squares method between computed an measured $T_1(t)$

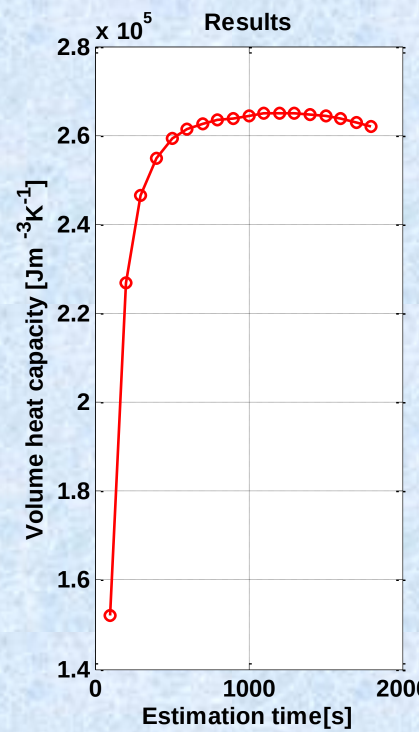
# III. Estimation with a pure thermal model

## 1. Case of a dry sample

- A wood fibre based insulator sample has been dried under vacuum




- ➢ The estimated curve perfectly fits the experimental data
- ➢ The estimated value $\lambda = 0.0512\ Wm^{-1}K^{-1}$ is in agreement with the manufacturer's value ($0.05\ Wm^{-1}K^{-1}$), the error is $< 3\%$
- ➢ The estimated value $\rho c = 265{,}000\ Jm^{-3}K^{-1}$ is coherent with the given density $\rho = 210\ kgm^{-3}$ and the measured $Cp = 1{,}274\ Jkg^{-1}K^{-1}$ (Setaram µDSC) which lead to $\rho c = 257{,}000\ Jm^{-3}K^{-1}$ (error $< 4\%$)

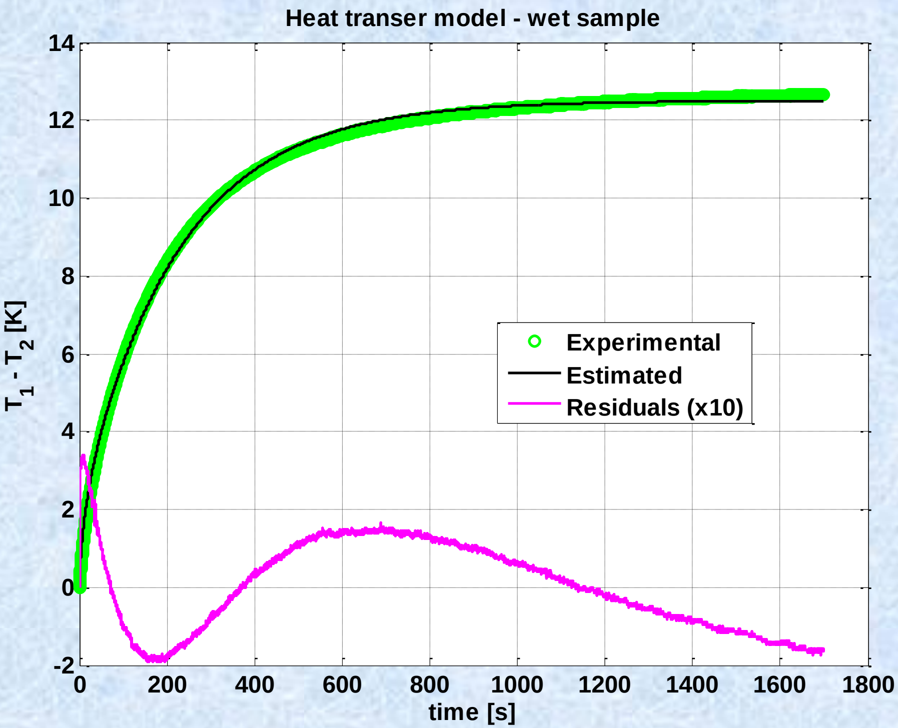


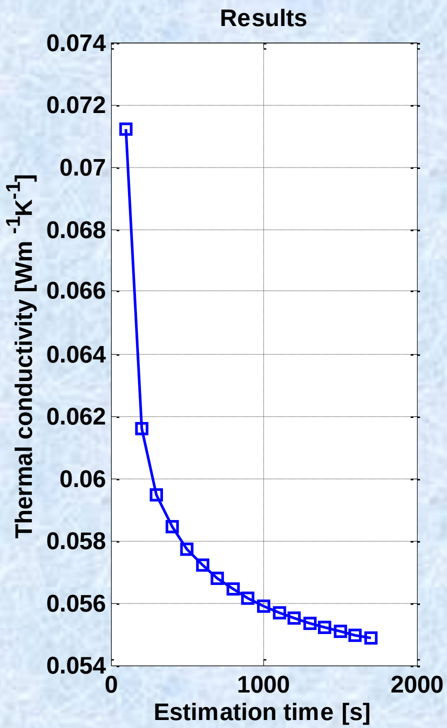
➢ The thermal model is suited to the dry sample

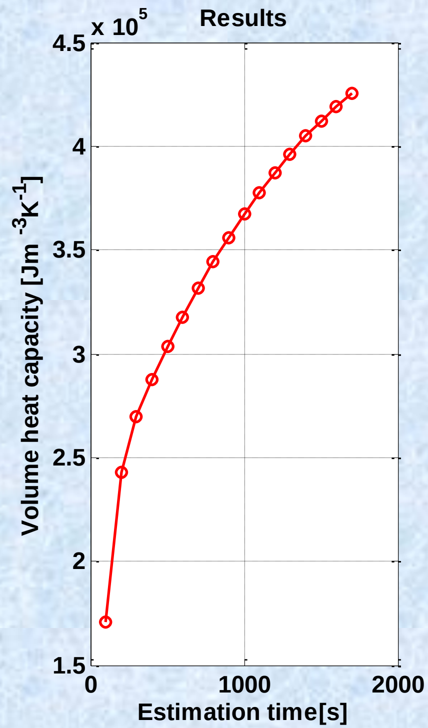
# III. Estimation with a pure thermal model

## 2. Case of a wet sample

- The same sample has been placed under ambient humidity conditions ($X = 6\%$)




- The residuals between the computed and experimental curves are signed
- The thermal conductivity is not reached even considering long times the last value is $\lambda = 0.055\ Wm^{-1}K^{-1}$ (10% error)
- The estimated value of the volume heat capacity also varies with the chosen time range and is very far from the real value



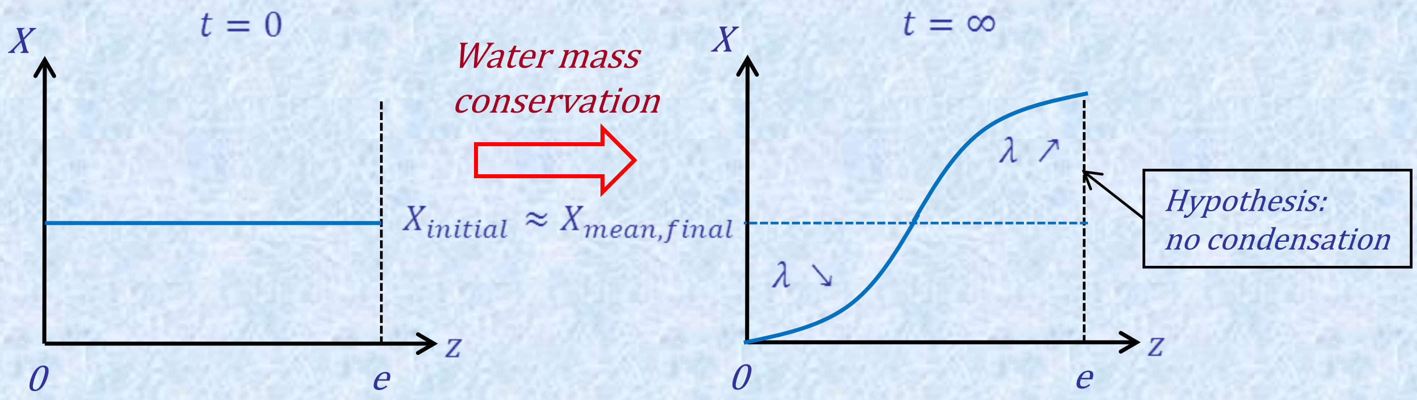
- The thermal model presents a bias for the wet sample

# III. Estimation with a pure thermal model

## 3. Hypothesis

What is the most pertinent way to process experimental data on a wet material from a hot plate method measurement with a pure thermal model ?

- 2 ideas are possible:
  - Processing only at short times: with the hypothesis that the thermal diffusion is much greater than the mass diffusion
  - Processing only at long times: with the hypothesis that once the thermal and mass equilibrium is reached, the water content X has not changed considering the whole thickness (only the profile has changed) and an heterogeneous equivalent thermal resistance can be deduced




- A coupled heat and mass transfer model for the hot plate method has to be developed to draw a conclusion

# IV. A coupled heat and mass transfer model

## 1. Modelling

Considering the coupled PDEs of conservation of heat and mass:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial z^2} + \frac{\rho_{dry}}{\rho c}\Delta H_v \frac{\partial X}{\partial t} \qquad \frac{\partial X}{\partial t} = D_m \frac{\partial^2 X}{\partial z^2} + \delta D_m \frac{\partial^2 T}{\partial z^2}$$

The boundary conditions for $z = 0$: $\Phi = \rho c_h \frac{\partial T}{\partial t} - \lambda \frac{\partial T}{\partial z}$ $\quad -D_m\left(\frac{\partial X}{\partial z} + \delta \frac{\partial T}{\partial z}\right) = 0$

The boundary conditions for $z = e$: $T = T_\infty$ $\quad -D_m\left(\frac{\partial X}{\partial z} + \delta \frac{\partial T}{\partial z}\right) = 0$

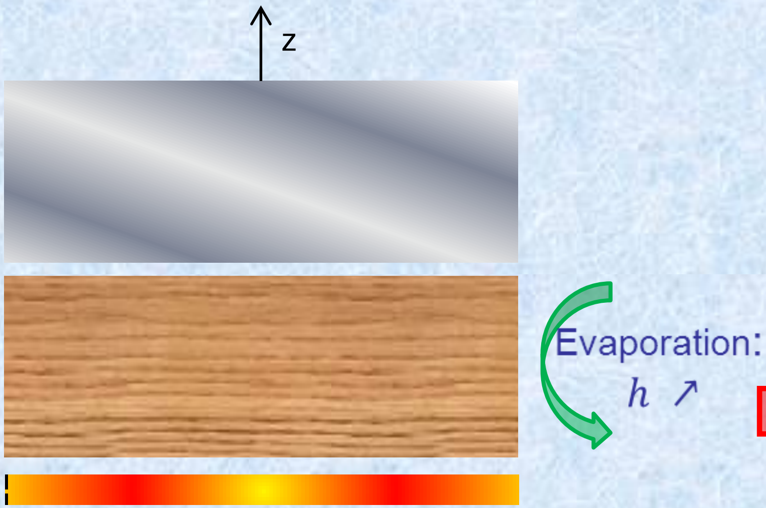
The initial conditions for t= 0: $T(z,0) = T_\infty$ $\quad X(z,0) = X_i$

- This system of coupled PDEs is solved using numerical methods (finite volumes)
  - $\rho c$ is a function of the water content $X$ and depends on $\rho c_{dry}$ which is given by the previous measurement on the dry sample $\quad \rho c(X) = \rho c_{dry}\left(1 + \frac{c_{water}}{c_{dry}} X\right)$
  - The capacity of the heating element $\rho c_h$ can also be deduced using the previously measured $mc_h$ and its dimensions.
- 3 unknown parameters are remaining: the thermal conductivity $\lambda$, the mass diffusion coefficient $D_m$ and the thermomigration coefficient $\delta$

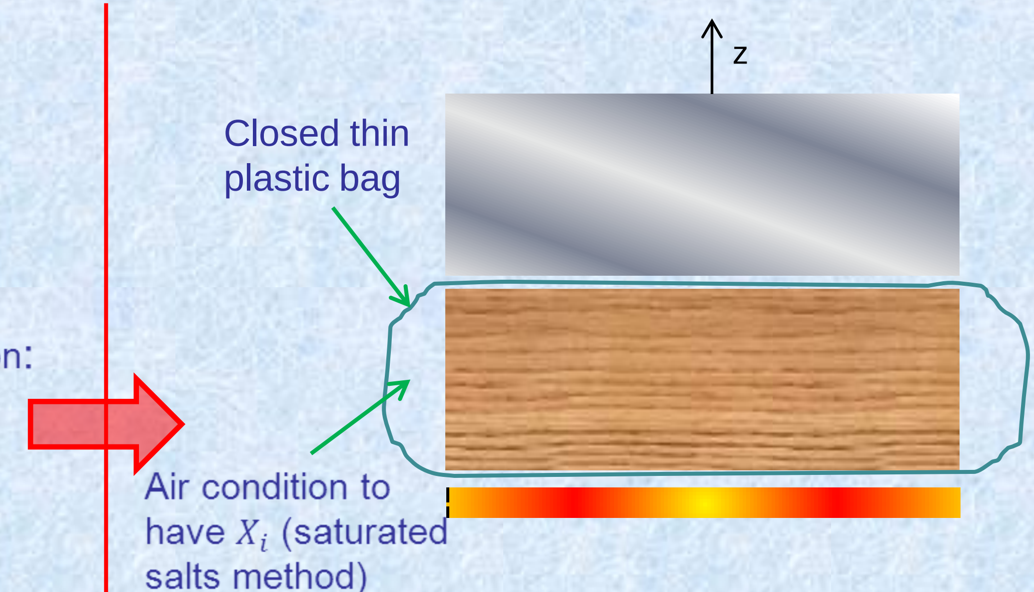
# IV. A coupled heat and mass transfer model

## 2. Experimental precautions

The hot plate experimentation on a sample with X% of water requires some precautions:




In this case, the initial conditions: $T(z,0) = T_\infty$ and $X(z,0) = X_i$ are not satisfied until the system is in equilibrium with the ambient air



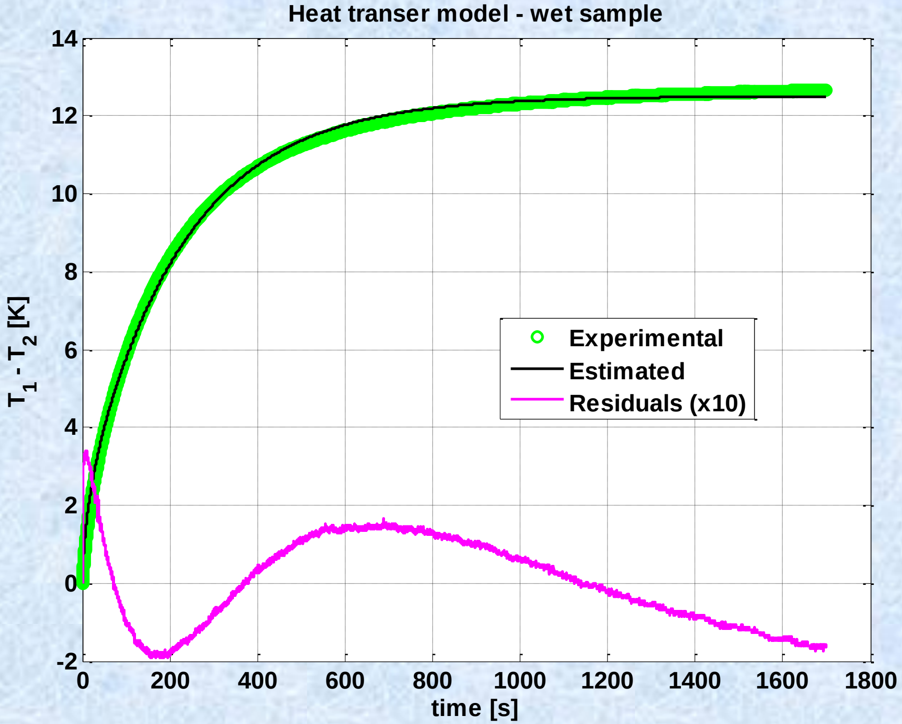

The sorption isotherm has been used to link the gas humidity and the water content X

- 3 unknown parameters are remaining: the thermal conductivity $\lambda$, the mass diffusion coefficient $D_m$ and the thermomigration coefficient $\delta$

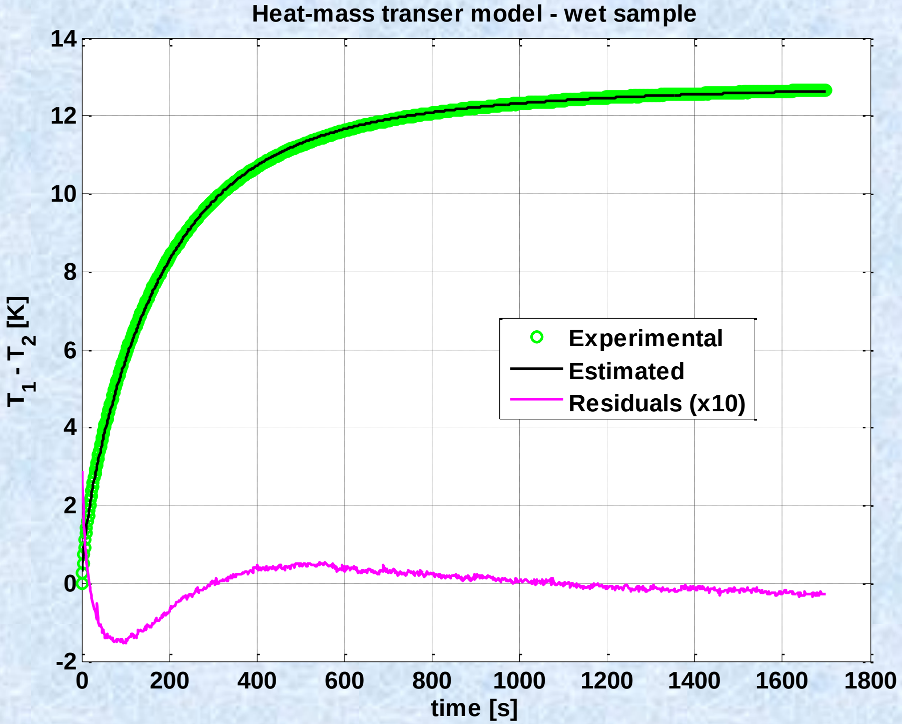
# IV. A coupled heat and mass transfer model

## 3. Experimental results

- The same sample has been placed under ambient humidity conditions ($X = 6\%$)




$\lambda = 0.055\ Wm^{-1}K^{-1}$

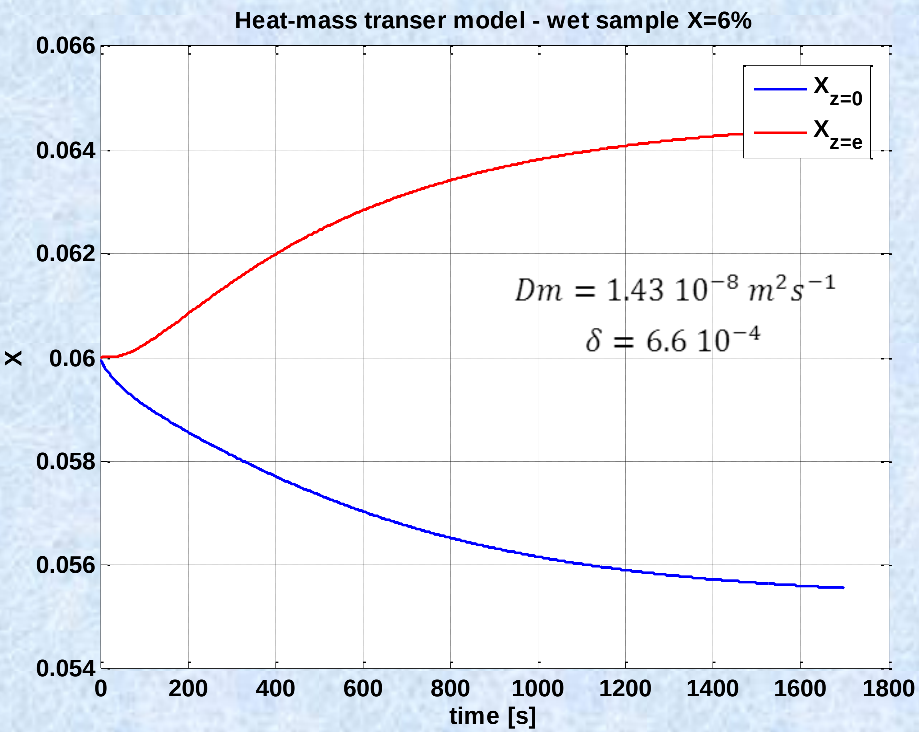


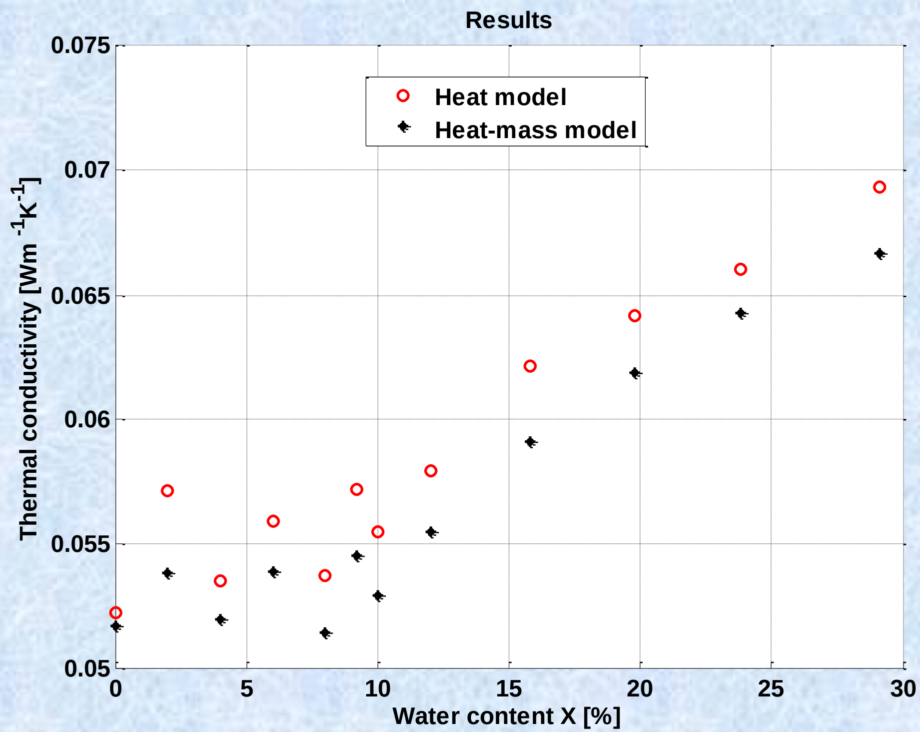

$\lambda = 0.0538\ Wm^{-1}K^{-1}$

- ➢ The residuals appear to be smoothed by the coupled heat-mass transfer model
- ➢ But a bias is still present, the model is improvable: double diffusion ?

# IV. A coupled heat and mass transfer model

## 3. Experimental results

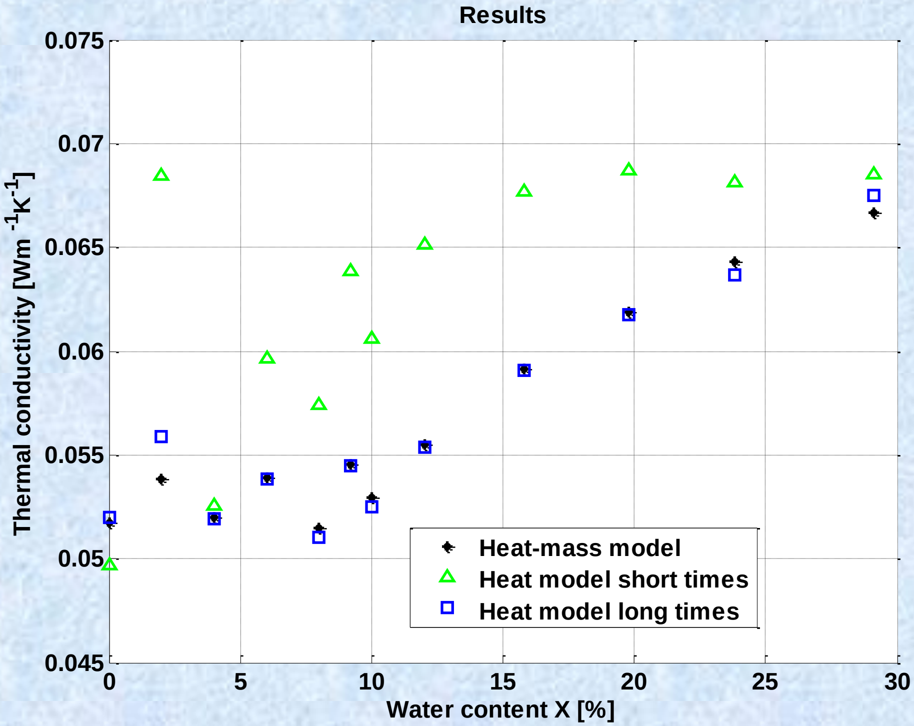



- The thermal conductivity seems to be overestimated by the pure thermal model
- The water content at the boundaries is realistic, decreasing at the heated surface and increasing at the unheated surface
- The heat and mass transfer model is used as a reference

# IV. A coupled heat and mass transfer model

## 3. Experimental results




- Processing hot plate measurements at short times is not recommended, the migration of the water immediately affects heat transfers into low density and permeable materials
- Thermal conductivity deduced from long times identification is reliable and coherent with the heat-mass transfer model: the water content profile has no significant influence on the thermal resistance assuming that there are no mass loss

# V. Conclusion

- Low density hygroscopic insulators have their thermal conductivity increasing with the ambient air humidity
- A method based on a pure thermal model may introduce an error
- Taking mass transfer into account is possible although it is improvable
- For the hot plate method, despite of the pure heat model, processing at long times can provide good results