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Measurement of the thermal conductivity of thin insulating anisotropic material with a stationary hot strip method

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Abstract
This paper presents a method dedicated to the thermal conductivity measurement of thin insulating anisotropic materials. The method is based on three hot-strip-type experiments in which the stationary temperature is measured at the center of the hot strip. A 3D model of the heat transfer in the system is established and simulated to determine the validity of a 2D transfer hypothesis at the center of the hot strip. A simplified 2D model is then developed leading to the definition of a geometrical factor calculable from a polynomial expression. A very simple calculation method enabling the estimation of the directional thermal conductivities from the three stationary temperature measurements and from the geometrical factor is presented. The uncertainties on each conductivity are estimated. The method is then validated by measurements on polyethylene foam and Ayous (anistropic low-density tropical wood); the estimated values of the thermal conductivities are in good agreement with the values estimated using the hot plate and the flash method. The method is finally applied on a thin super-insulating fibrous material for which no other method is able to measure the in-plane conductivity.

Keywords: thermal conductivity, anisotropy, thin insulating material, hot strip, stationary method

(Some figures in this article are in colour only in the electronic version)

Nomenclature

a	thermal diffusivity ($m^2\ s^{-1}$)
b	half width of the hot strip (m)
c	half length of the hot strip (m)
c_s	specific heat of the sample ($J\ K^{-1}\ kg^{-1}$)
d	half width of the sample (m)
e	half length of the sample (m)
e_s	half thickness of the hot strip (m)
f	thickness of the sample (m)
h	global heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
F	geometric factor
N	number of experimental points
S	sample and heating element area (m^2)
$T(x, y, z, t)$	sample temperature (°C)
T_i	initial temperature of the system (°C)
T_0	stationary temperature at the center of the hot strip (°C)
ϕ_0	half heat flux density in the heating element ($W\ m^{-2}$)
λ	thermal conductivity of the sample ($W\ m^{-1}\ K^{-1}$)
ρ_s	density of the hot strip ($kg\ m^{-3}$)

Subscripts

1, 2, 3	experiment number
exp	experimental

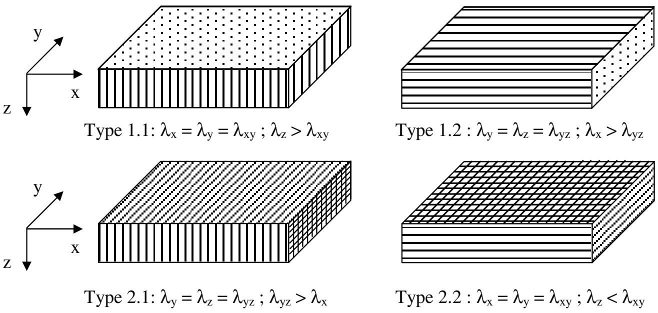

Figure 1. Schema of samples of types 1 and 2.

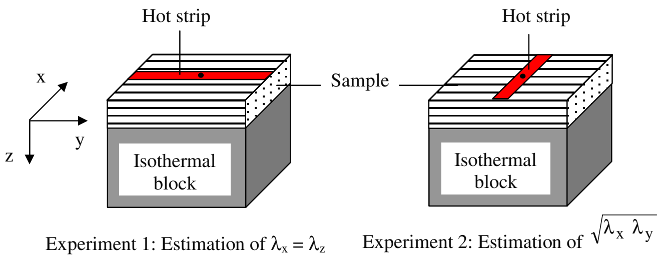

Figure 2. Schema of the two experiments enabling the estimation of $\lambda_y = \lambda_z$ and λ_x with the hot strip method (Ladevie *et al* 2000).

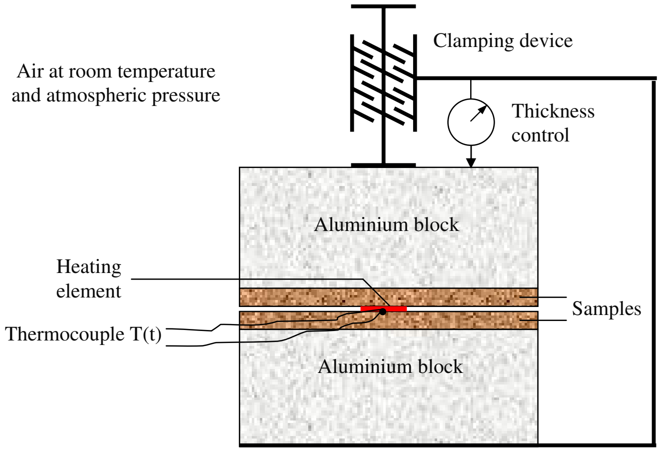

Figure 3. Schema of the experimental device.

1. Introduction

Many insulating materials are anisotropic, particularly fibrous materials. These fibers whose thermal conductivity is greater than the conductivity of the surrounding material (most commonly air) create preferential paths for heat flow. Moreover, the thermal conductivity of the composite fibrous material may be anistropic if the fibers are oriented in only one or two directions.

Fibrous materials may be classified into three types.

- The materials of type 1 containing fibers oriented in only one direction such as wood.
- The materials of type 2 containing fibers oriented in two directions such as tissue.
- The materials of type 3 containing fibers oriented in directions that are not parallel to the face of the sample. These materials have three different directional thermal conductivities.

The materials of the first two types may be available in a thin sheet with one of the two configurations represented in figure 1.

The measurement of the two or three directional thermal conductivities of anisotropic materials, available only in a thin sheet, is complex.

- The methods of the guarded hot plate (Salmon 2001, Xaman *et al* 2009, Huang 2006), of the three-layer device (Jannot *et al* 2009) and of the tiny hot plate (Jannot *et al* 2010b) make only the measurement of the conductivity λ_z possible in the direction perpendicular to the heating element.
- The methods of the hot wire (Andersson 1976, Zhang *et al* 1993, Rharbaoui 1994, Coquard *et al* 2006) and of the hot disk (Gustafsson 1991, Gustavsson *et al* 1994, He 2005) are based on the semi-infinite hypothesis and so are not applicable with low thickness samples.
- The flash method (Degiovanni 1977) makes only the measurement of the thermal diffusivity *a* possible.
- The laser flash method (Cernuschi *et al* 2004) cannot be applied to insulating materials.

The hot strip method with the temperature of the rear face of the sample maintained constant (Ladevie *et al* 2000) makes the measurement of two directional thermal conductivities (with a 90° rotation of the sample between the two measurements, cf figure 2) possible for a material of type 1.2 or 2.1. An approach has been realized (Gobbé *et al* 2004) to adapt the method for materials of type 1.1 or 2.2. But the validation has been limited to the estimation of the conductivity $\lambda_x = \lambda_y$ of a bi-layer material from a hot strip measurement and previous knowledge of the thermal conductivity λ_z.

A method enabling the estimation of the directional thermal conductivities λ_x, λ_y and λ_z of an anisotropic thin insulating material in a more general case is presented as follows.

2. Experimental device

The experimental device is shown in figure 3. Since it is symmetrical, all of the other schematics will show only a half view of the actual device. It is composed of the following.

- A heating plane strip called hot strip with a low thickness (0.25 mm) on which a type K thermocouple is fixed. It is placed between two samples of the material to be characterized. Since the tested insulating materials are not totally rigid, a clamping pressure produces a local surface deformation that is sufficient to insert the thermocouple in the material and thus eliminate uneven thermal contact resistance.
- Two isothermal blocks, made of aluminum, with 4 cm thickness and the same section as the samples.
- A tightening device enabling control of the pressure and of the thickness of the device placed between the

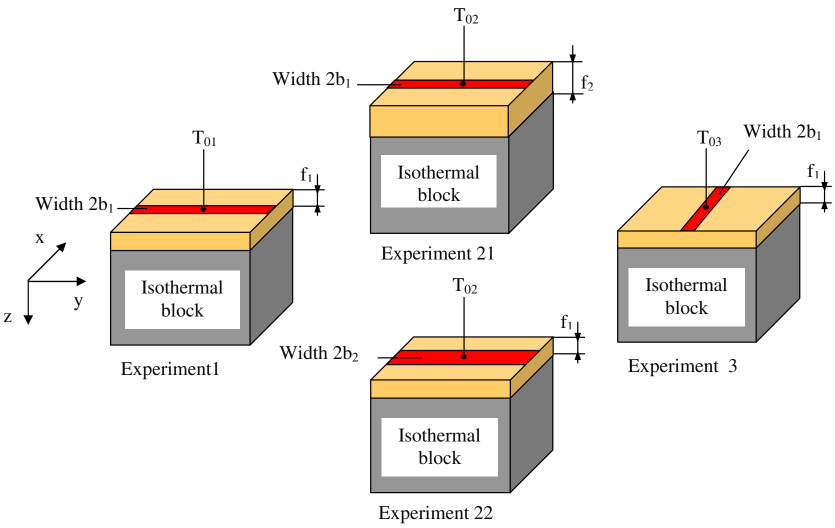

Figure 4. Schema of the method based on three measurements.

aluminum blocks. The pressure value is increased until the samples' thickness begins to decrease; this depends on their pressure resistance. Since pressure is applied, no additional material such as paste is needed to improve thermal contacts.

A heat flux step is produced in the hot strip, and the temperature T_0 that is reached in the stationary regime at the center of the hot strip is measured. T_0 is measured when its variation is less than 0.1 °C after 30 min, which is low compared to its final temperature value of around 10 °C. The (uniform) temperature of each aluminum block is measured to make sure that it remains constant.

The principle of the proposed method is to realize three successive measurements (cf figure 4).

- A measurement of T_{01} with a hot strip with width $2b_1$ and two samples with the same thickness f_1.
- A measurement of T_{02} with either a hot strip with width $2b_1$ and two samples with the same thickness $f_2 \neq f_1$, or a hot strip with width $2b_2 \neq 2b_1$ and two samples with the same thickness f_1.
- A measurement of T_{03} with a hot strip with width $2b_1$ and two samples with the same thickness f_1 after the hot strip has been rotated by 90°.

The thermal conductivities λ_y and λ_z may be estimated from the measured values of T_{01} and T_{02} if one knows

- the heat flux density $2\phi_0$ produced by the Joule effect in the hot strip;
- either the width $2b_1$ of the strip and thicknesses f_1 and f_2 of the samples, or the widths $2b_1$ and $2b_2$ of the strips and the thickness f_1 of the samples.

In the same way, if the thermal conductivity λ_z is known, the measured value of T_{03} enables the estimation of the thermal conductivity λ_x.

The relation between equilibrium temperature and thermal conductivities is not linear since in the case of the measurement of T_{01} or T_{02},

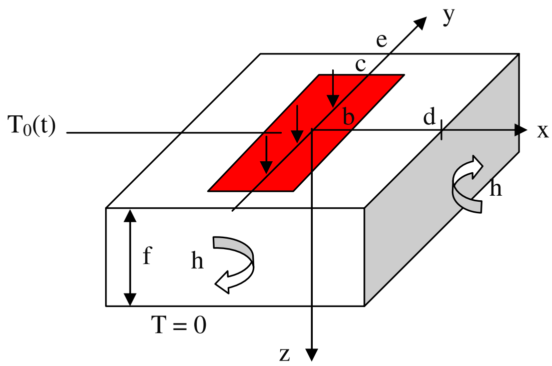

Figure 5. Schema of the modeled system.

- if f is very small (or if b is very large), the equilibrium temperature only depends on λ_z.
- if f is very large (or if b is very small), the equilibrium temperature only depends on $\sqrt{\lambda_y \lambda_z}$.

Modeling the system represented in figure 5 will show more precisely how the thermal conductivities λ_y and λ_z may be calculated from the measured values of T_{01} and T_{02} and then how the thermal conductivity λ_x may be calculated from the measured values of T_{03}.

3. Modeling

3.1. Complete model

The sample dimensions are width $2d$, length $2e$ and thickness f. The hot strip dimensions are thickness $2e_s$, width $2b$, length $2c$, and its density is ρ_s and specific heat c_s.

The hypotheses are as follows.

- The thermal resistance of the hot strip is null (thin element).
- The thermal contact resistances are negligible compared to the thermal resistance of the sample.
- The temperature of the unheated face remains constant.

- Thermal radiation inside the samples is negligible.

The first two hypotheses will be verified if the thermal resistance of the sample is one hundred times greater than the sum of the thermal resistance of the strip and the two thermal contact resistances (between the sample and the heating element and between the sample and the aluminum block). The half thickness of the heating element is 0.1 mm; its thermal conductivity is around 0.25 W K^{-1} m^{-1} so that its thermal resistance is 4×10^{-4} K W^{-1} m^2. If it is assumed that the thermal contact resistances are lower than 2×10^{-4} K W^{-1} m^2, then the hypothesis is verified if the thermal resistance f/λ of the sample (f being its thickness) is greater than 8×10^{-2} K W^{-1} m^2.

The heat transfer equation is

$$\lambda_x \frac{\partial^2 T(x,y,z,t)}{\partial x^2} + \lambda_y \frac{\partial^2 T(x,y,z,t)}{\partial y^2} + \lambda_z \frac{\partial^2 T(x,y,z,t)}{\partial z^2} = \rho c \frac{\partial T(x,y,z,t)}{\partial t}. \tag{1}$$

The initial and boundary conditions are

$$T(x,y,z,0) = T_i \tag{2}$$

$$\frac{\partial T(0,y,z,t)}{\partial x} = 0 \tag{3}$$

$$\frac{\partial T(x,0,z,t)}{\partial y} = 0 \tag{4}$$

$$-\lambda_x \frac{\partial T(d,y,z,t)}{\partial x} = h[T(d,y,z,t) - T_i] \tag{5}$$

$$-\lambda_y \frac{\partial T(x,e,z,t)}{\partial y} = h[T(x,e,z,t) - T_i] \tag{6}$$

$$T(x,y,f,t) = 0 \tag{7}$$

$$\phi_0(x,y,t) = \rho_s e_s c_s \frac{\partial T(x,y,0,t)}{\partial t} - \lambda_z \frac{\partial T(x,y,0,t)}{\partial z}, \tag{8}$$

where $\phi_0\,(x,y,t) = \phi_0$ if $x \leqslant b$ and $y \leqslant c$, and $\phi_0\,(t) = 0$ if $x > b$ and $y > c$; h is the global (convection + radiation) heat transfer coefficient.

Since the thermal contact resistance and the thermal resistance of the hot strip are neglected, the temperature of the hot strip and the temperature of the sample at $z = 0$ are identical. Set $\Delta T(x,y,z,t) = T(x,y,z,t) - T_i$ and $L\,[\Delta T(x,y,z,t)] = \theta(x,y,z,p)$.

The Laplace transform of relation (1) may be written as

$$\lambda_x \frac{\partial^2 \theta(x,y,z,p)}{\partial x^2} + \lambda_y \frac{\partial^2 \theta(x,y,z,p)}{\partial y^2} + \lambda_z \frac{\partial^2 \theta(x,y,z,p)}{\partial z^2} = \rho c\; p\theta(x,y,p). \tag{9}$$

Using the separation of variables method, the Laplace transform of the temperature may be written as

$$\theta(x,y,z,p) = X(x,p)Y(y,p)Z(z,p).$$

The general solution of the system of equations (1) to (7) is

$$\theta(x,y,z,p) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} A_{pq}\cos(\alpha_p x) \times \cos(\delta_q z)\sinh[\gamma_{pq}(f-y)]. \tag{10}$$

The non-homogeneous boundary relation (8) is then used to estimate the coefficients A_{pq}, leading to

$$A_{pq} = \frac{\Phi_0(p)\frac{\sin(\alpha_p b)}{\alpha_p}\frac{\sin(\delta_q c)}{\delta_q}}{[\lambda_z\,\gamma_{pq}\cosh(\gamma_{pq} f) + \rho_s c_s e_s p \sinh(\gamma_{pq} f)]\left[\frac{\sin(2\alpha_p d)}{4\alpha_p} + \frac{d}{2}\right]\left[\frac{\sin(2\,\delta_q e)}{4\,\delta_q} + \frac{e}{2}\right]}. \tag{11}$$

The Laplace transform of the temperature at the center of the heating element is

$$\theta(0,0,0,p) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \frac{\Phi_0(p)\frac{\sin(\alpha_p b)}{\alpha_p}\frac{\sin(\delta_q c)}{\delta_q}\sinh(\gamma_{pq}\,f)}{F_{pq}\left[\frac{\sin(2\alpha_p d)}{4\alpha_p} + \frac{d}{2}\right]\left[\frac{\sin(2\delta_q e)}{4\delta_q} + \frac{e}{2}\right]}. \tag{12}$$

The eigenvalues α_p are the solutions of the equation

$$\alpha d \tan(\alpha d) = H_x \tag{13}$$

with

$$H_x = \frac{hd}{\lambda_x}. \tag{14}$$

The eigenvalues δ_q are the solutions of the equation

$$\delta e \tan(\delta e) = H_y \tag{15}$$

with

$$H_z = \frac{he}{\lambda_y}. \tag{16}$$

α, δ and γ are linked by the relation

$$\lambda_z \gamma^2 = \rho c p + \lambda_x \alpha^2 + \lambda_y \delta^2. \tag{17}$$

If the heat flux is a flux step, then

$$\Phi_0\,(p) = \frac{\phi_0}{p}.$$

This model of the transient regime will enable us to verify *a posteriori* that the stationary regime is reached when the temperatures T_{0i} at the center of the hot strip are measured.

The asymptotic solution in the stationary regime is

$$T(0,0) = T_0 = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \frac{\phi_0\frac{\sin(\alpha_p b)}{\alpha_p}\frac{\sin(\delta_q c)}{\delta_q}\sinh(\gamma_{pq} f)}{\lambda_z\gamma_{pq}\cos h(\gamma_{pq} f)\left[\frac{\sin(2\alpha_p d)}{4\alpha_p} + \frac{d}{2}\right]\left[\frac{\sin(2\delta_q e)}{4\delta_q} + \frac{e}{2}\right]}. \tag{18}$$

A first method might consist in solving numerically the following system of three equations with three unknown parameters:

$$T_{01} = F\left(f_1, \lambda_x, \lambda_y, \lambda_z, b\right) \tag{19}$$

$$T_{02} = F\left(f_2, \lambda_x, \lambda_y, \lambda_z, b\right) \tag{20}$$

$$T_{03} = F\left(f_1, \lambda_x, \lambda_y, \lambda_z, b\right), \tag{21}$$

where T_{01}, T_{02} and T_{03} are calculated by relation (18) considering $h = 10$ W m^{-2} K^{-1}. The estimation of the parameters λ_y, λ_y and λ_z may be realized by using an iterative

Table 1. Values of the coefficients of the polynomial I.

a_6	a_5	a_4	a_3	a_2	a_1	a_0
-2.7042×10^{-4}	0.005 537 5	−0.047 516 4	0.224 43	−0.655 252	1.3614	−0.060 99

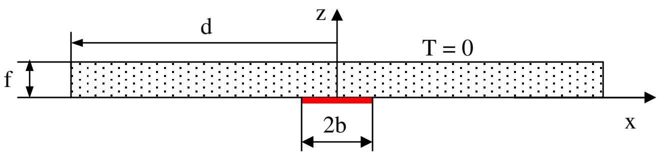

Figure 6. Schema of the simplified 2D model.

method based on the minimization by the Levenberg–Marquart algorithm of the following sum Σ (nonlinear least squares):

$$\Sigma = [T_{01\,\mathrm{exp}} - F(f_1, \lambda_x, \lambda_y, \lambda_z, b)]^2 + [T_{02\,\mathrm{exp}} - F(f_2, \lambda_x, \lambda_y, \lambda_z, b)]^2 + [T_{03\,\mathrm{exp}} - F(f_1, \lambda_x, \lambda_y, \lambda_z, b)]^2. \quad (22)$$

The problem is that the function F is not an explicit function of λ_y, λ_y and λ_z; its calculation needs the resolution of the two equations (13) and (15) and the successive calculations of two infinite sums as shown by relation (18).

3.2. Simplified model

Another method consists in using an explicit function G established from a simplified 2D model instead of the complex function F. It would enable a faster and easier estimation of the thermal conductivities. The system represented in figure 6 has been modeled.

The following hypotheses are considered.

- The length of the hot strip is sufficiently long compared to its width so that the center temperature does not depend on y.
- The width of the sample is large enough so that it can be considered as a semi-infinite medium in the Ox direction.

The system verifies the following equations:

$$\lambda_x \frac{\partial^2 T(x,z)}{\partial x^2} + \lambda_z \frac{\partial^2 T(x,z)}{\partial z^2} = 0 \quad (23)$$

$$T(x, f) = 0 \quad (24)$$

$$\frac{\partial T(0,z)}{\partial x} = 0 \quad (25)$$

$$\lim_{x\to\infty} \frac{\partial T}{\partial x} = 0 \quad (26)$$

$$\text{if } x < b: \quad \phi_0 = -\lambda_z \frac{\partial T(x,0)}{\partial z} \quad (27)$$

$$\text{if } x > b: \quad \phi_0 = 0. \quad (28)$$

Using the variables separation method leads to the solution

$$T(0,0) = \frac{2\phi_0}{\pi\sqrt{\lambda_x\lambda_z}} \int_0^\infty \frac{\sin(\alpha b)}{\alpha^2} \tanh\left(\sqrt{\frac{\lambda_x}{\lambda_z}}\alpha f\right) \mathrm{d}\alpha. \quad (29)$$

It may also be written as

$$T(0,0) = \frac{b\phi_0}{\sqrt{\lambda_x\lambda_z}} I(u) \quad \text{with} \quad u = \sqrt{\frac{\lambda_x}{\lambda_z}}\frac{f}{b} \quad (30)$$

where

$$I(u) = \frac{2}{\pi}\int_0^\infty \frac{\sin(\omega)}{\omega^2}\tanh(u\omega)\,\mathrm{d}\omega. \quad (31)$$

The function I may be represented with a precision better than 0.15% for $u \in [0.5, 5]$ by the following polynomial:

$$I(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 + a_6 u^6. \quad (32)$$

The values of the coefficients a_i determined for $u \in [0.5, 5]$ are given in table 1.

If the hypotheses of this simplified model are verified, the thermal conductivities λ_x and λ_z may be easily deduced from the measured values of two equilibrium temperatures: T_{01} for a heating element with width $2b_1$ and two samples of thickness f_1, and T_{02} for a heating element with width $2b_2$ and two samples of thickness f_2. The following simple system must be solved:

$$T_{01_x} = \frac{b_1\phi_1}{\sqrt{\lambda_x\lambda_z}} I\left(\sqrt{\frac{\lambda_x}{\lambda_z}}\frac{f_1}{b_1}\right) \quad (33)$$

$$T_{02_z} = \frac{b_2\phi_2}{\sqrt{\lambda_x\lambda_z}} I\left(\sqrt{\frac{\lambda_x}{\lambda_z}}\frac{f_2}{b_2}\right), \quad (34)$$

where I is a polynomial function given by relation (32).

Relations (33) and (34) established from the simplified 2D model highlight the symmetrical influence of the parameters b and f. It is thus theoretically possible to estimate separately λ_x and λ_z from two experiments:

- either using two hot strips of respective widths b_1 and b_2 with two samples having the same thickness f,
- or using a unique hot strip of width b and two couples of samples of thickness f_1 and f_2 respectively.

The limit cases are as follows.

- A very large width hot strip corresponding to a hot plate with a center temperature measurement enabling direct estimation of λ_z.
- A very large thickness sample corresponding to a semi-infinite medium for which the temperature of the hot strip tends toward the temperature of the hot wire and from which $\sqrt{\lambda_x\lambda_z}$ can be estimated.

After the results of experiments 1 and 21 or 22 (cf figure 4) have been used to estimate the thermal conductivities λ_z and λ_x, the result T_{03} of experiment 3 is used to estimate the thermal conductivity λ_y by solving the equation

$$T_{03} = \frac{b_1\phi_3}{\sqrt{\lambda_y\lambda_z}} I\left(\sqrt{\frac{\lambda_y}{\lambda_z}}\frac{f_1}{b_1}\right) \quad (35)$$

in which the only unknown parameter is λ_y.

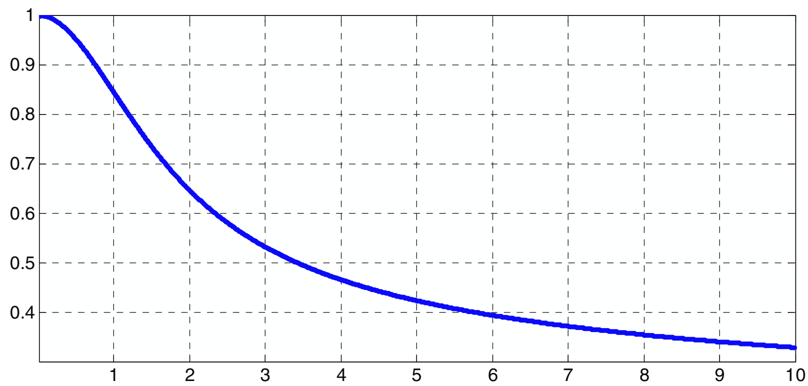

Figure 7. Function $\alpha(u)$.

3.3. Optimal configuration

Relations (33) and (34) enable us to write

$$\log(T_{01}) = \log(b_1) + \log(\phi_1) - \log(K_1) + \log[I(u_1)] \quad (36)$$

$$\log(T_{02}) = \log(b_2) + \log(\phi_2) - \log(K_1) + \log[I(u_2)], \quad (37)$$

where $K_1 = \sqrt{\lambda_x \lambda_z}$; $K_2 = \sqrt{\frac{\lambda_x}{\lambda_z}}$; $u_i = K_2 \frac{f_i}{b_i}$.

Solving the system obtained by derivation of relations (36) and (37) leads to

$$\begin{aligned}(\alpha_2 - \alpha_1)\frac{\mathrm{d}\lambda_x}{\lambda_x} &= (1+\alpha_1)\frac{\mathrm{d}T_{02}}{T_{02}} - (1+\alpha_2)\frac{\mathrm{d}T_{01}}{T_{01}} \\ &+ (1+\alpha_2)\frac{\mathrm{d}\phi_1}{\phi_1} - (1+\alpha_1)\frac{\mathrm{d}\phi_2}{\phi_2} \\ &+ (1+\alpha_1)(1+\alpha_2)\left(\frac{\mathrm{d}b_1}{b_1} - \frac{\mathrm{d}b_2}{b_2}\right) \\ &+ (1+\alpha_2)\alpha_1\frac{\mathrm{d}f_1}{f_1} - (1+\alpha_1)\alpha_2\frac{\mathrm{d}f_2}{f_2} \quad (38)\end{aligned}$$

$$\begin{aligned}(\alpha_2 - \alpha_1)\frac{\mathrm{d}\lambda_z}{\lambda_z} &= (1-\alpha_2)\frac{\mathrm{d}T_{01}}{T_{01}} - (1-\alpha_1)\frac{\mathrm{d}T_{02}}{T_{02}} \\ &+ (1-\alpha_1)\frac{\mathrm{d}\phi_2}{\phi_2} - (1-\alpha_2)\frac{\mathrm{d}\phi_1}{\phi_1} \\ &+ (1-\alpha_1)(1-\alpha_2)\left(\frac{\mathrm{d}b_2}{b_2} - \frac{\mathrm{d}b_1}{b_1}\right) \\ &- (1-\alpha_2)\alpha_1\frac{\mathrm{d}f_1}{f_1} + (1-\alpha_1)\alpha_2\frac{\mathrm{d}f_2}{f_2}, \quad (39)\end{aligned}$$

where $\alpha_i = \alpha(u_i) = u_i \frac{\partial[\ln(I(u_i))]}{\partial u_i}$; figure 7 represents the curve $\alpha(u)$ calculated numerically. It decreases from $\alpha(0) = 1$ to $\alpha(u \to \infty) = 0$.

Since all the parameters are independent, the maximum error can be calculated by replacing each term by its absolute value:

$$\begin{aligned}\frac{\Delta\lambda_x}{\lambda_x} &= \frac{1+\alpha_1}{\alpha_2-\alpha_1}\left(\frac{\Delta T_{02}}{T_{02}} + \frac{\Delta\phi_2}{\phi_2}\right) + \frac{1+\alpha_2}{\alpha_2-\alpha_1}\left(\frac{\Delta T_{01}}{T_{01}} + \frac{\Delta\phi_1}{\phi_1}\right) \\ &+ \frac{(1+\alpha_1)(1+\alpha_2)}{\alpha_2-\alpha_1}\left(\frac{\Delta b_1}{b_1} + \frac{\Delta b_2}{b_2}\right) \\ &+ \frac{(1+\alpha_2)\alpha_1}{\alpha_2-\alpha_1}\frac{\Delta f_1}{f_1} + \frac{(1+\alpha_1)\alpha_2}{\alpha_2-\alpha_1}\frac{\Delta f_2}{f_2} \quad (40)\end{aligned}$$

$$\begin{aligned}\frac{\Delta\lambda_z}{\lambda_z} &= \frac{1-\alpha_2}{\alpha_2-\alpha_1}\left(\frac{\Delta T_{01}}{T_{01}} + \frac{\Delta\phi_1}{\phi_1}\right) \\ &+ \frac{1-\alpha_1}{\alpha_2-\alpha_1}\left(\frac{\Delta T_{02}}{T_{02}} + \frac{\Delta\phi_2}{\phi_2}\right) \\ &+ \frac{(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left(\frac{\Delta b_2}{b_2} + \frac{\Delta b_1}{b_1}\right) \\ &+ \frac{(1-\alpha_2)\alpha_1}{\alpha_2-\alpha_1}\frac{\Delta f_1}{f_1} + \frac{(1-\alpha_1)\alpha_2}{\alpha_2-\alpha_1}\frac{\Delta f_2}{f_2}. \quad (41)\end{aligned}$$

The analysis of the expressions $\frac{1+\alpha_1}{\alpha_2-\alpha_1}$, $\frac{1+\alpha_2}{\alpha_2-\alpha_1}$, $\frac{(1+\alpha_1)(1+\alpha_2)}{\alpha_2-\alpha_1}$, $\frac{(1+\alpha_2)\alpha_1}{\alpha_2-\alpha_1}$, $\frac{(1+\alpha_1)\alpha_2}{\alpha_2-\alpha_1}$, $\frac{1-\alpha_2}{\alpha_2-\alpha_1}$, $\frac{1-\alpha_1}{\alpha_2-\alpha_1}$, $\frac{(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}$, $\frac{(1-\alpha_2)\alpha_1}{\alpha_2-\alpha_1}$, $\frac{(1-\alpha_1)\alpha_2}{\alpha_2-\alpha_1}$ shows that they are all increasing with α_1 and decreasing with α_2. The minimum error on λ_x and λ_z is thus obtained for the minimum value of α_1 and for the maximum value of α_2. From figure 7, one can deduce that the optimal solution is to realize a centered hot plate experiment ($u_2 = 0$) whose limits have already been studied (Jannot *et al* 2010a) and an experiment conducted with a low width hot strip or with a large thickness sample ($u_1 \to \infty$).

In this optimal configuration ($\alpha_1 = 0$ and $\alpha_2 = 1$), the uncertainties on the estimated thermal conductivities are

$$\begin{aligned}\frac{\Delta\lambda_x}{\lambda_x} &= \left(\frac{\Delta T_{02}}{T_{02}} + \frac{\Delta\phi_2}{\phi_2}\right) + 2\left(\frac{\Delta T_{01}}{T_{01}} + \frac{\Delta\phi_1}{\phi_1}\right) \\ &+ 2\left(\frac{\Delta b_1}{b_1} + \frac{\Delta b_2}{b_2}\right) + \frac{\Delta f_2}{f_2} \quad (42)\end{aligned}$$

$$\frac{\Delta\lambda_z}{\lambda_z} = \left(\frac{\Delta T_{02}}{T_{02}} + \frac{\Delta\phi_2}{\phi_2}\right) + \frac{\Delta f_2}{f_2}. \quad (43)$$

As an example, within the hypotheses $\frac{\Delta T_{01}}{T_{01}} = \frac{\Delta T_{02}}{T_{02}}$, $\frac{\Delta\phi_1}{\phi_1} = \frac{\Delta\phi_2}{\phi_2}$, $\frac{\Delta b_1}{b_1} = \frac{\Delta f_1}{f_1} = \frac{\Delta f_2}{f_2}$ and $\frac{\Delta b_2}{b_2} = 0$ (since $b_2 \to \infty$), the uncertainties are linked by the relation

$$\frac{\Delta\lambda_x}{\lambda_x} = 3\frac{\Delta\lambda_z}{\lambda_z} \quad (44)$$

with $\frac{\Delta\lambda_z}{\lambda_z} = \frac{\Delta T_0}{T_0} + \frac{\Delta\phi}{\phi} + \frac{\Delta f}{f}$.

The uncertainty on λ_y is the same as on λ_x since experiments 1 and 3 are realized with the same hot strip and same samples. In the other cases, the ratio between $\frac{\Delta\lambda_x}{\lambda_x}$ and $\frac{\Delta\lambda_z}{\lambda_z}$ is always greater than 3. In practice, it is necessary to use relations (40) and (41) as will be done for the experimental results further presented; this will lead to a ratio greater than 3 as expected.

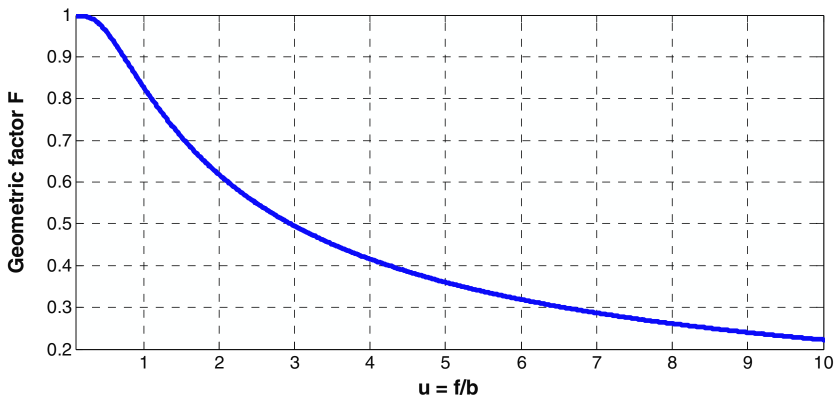

Figure 8. Value of the geometrical factor F.

3.4. Isotropic material

In the case of an isotropic material, the thermal conductivity can be calculated explicitly using the relation

$$\lambda = \frac{f\phi_0}{T_0} F\left(\frac{f}{b}\right), \tag{45}$$

where

- $F\left(\frac{f}{b}\right)$ is a geometrical factor defined by $F(u) = \frac{I(u)}{u}$. It is calculable directly using relation (32) or by numerical calculation of relation (31); its graphical representation is given in figure 8.
- ϕ is the heat flux density produced in the hot strip.
- T_0 is the stationary temperature reached at the center of the hot strip.
- b and f are respectively the half width and the thickness of the sample.

3.5. Validity domain of the simplified model

The simplified 2D model is based on the following hypotheses.

- The hot strip is long enough so that the heat transfer at its extremities has no influence on the center temperature (T_{01} and T_{02} do not depend on λ_y in experiments 1 and 2, and T_{03} does not depend on λ_x in experiment 3, cf figure 4).
- The sample is wide enough so that the convective lateral heat transfer has no influence on the center temperature.

The validity limits of these hypotheses will be defined considering that the measurement is realized with isotropic samples. The samples have a square cross-section with width equal to the length of the hot strip ($c = d = e$). The convective heat transfer coefficient on the faces defined by $y = e$ and $x = d$ (cf figure 5) is fixed at $h = 10$ W m^{-2} K^{-2}, and the blocks are supposedly perfectly isothermal. The width of the hot strip will be fixed firstly to $2b = 6$ mm and then to $2b = 12$ mm.

For various values of λ, the maximum value of the sample thickness leading to an estimation error of 1% is calculated. For each case considered, a complete 3D simulation using relation (18) is realized to calculate the stationary value of T_0 that is then used to estimate λ using relation (31). The deviation between the nominal and the estimated value is then calculated.

As an example, figure 9 represents the results obtained for sample with a square cross-section of 40×40 mm^2.

One can note that for a hot strip with width 6 mm and length 40 mm, the simplified model is valuable for thickness up to 9 mm for super-insulating materials and up to 15 mm for material with conductivities around 0.25 W m^{-1} K^{-1}. With the proposed simplified method, it is thus not necessary to get large cross-section samples to measure the thermal conductivity of the material whose thickness is less than 10 mm.

In the case of a heating element with other dimensions or of an anisotropic material, the simplified method may be used to estimate λ_x, λ_y and λ_z. Then, relations (19)–(21) must be used to calculate T_{01}, T_{02} and T_{03} and to verify that their deviation from the values calculated with relations (33)–(35) are negligible.

4. Experimental results and discussion

An experimental study has been carried out with three materials.

- An isotropic low-density material: polyethylene foam. Its thermal conductivity measured by the three-layer device (Jannot *et al* 2009) is $\lambda = 0.0425$ W m^{-1} K^{-1} with an estimated precision of 5%.
- An anisotropic material: Ayous, which is a low-density tropical wood. Three hot plate measurements realized in three different directions enable the estimation of the thermal effusivities E_x, E_y and E_z. A measurement with the flash method on a 6 mm thickness sample enables the estimation of the thermal diffusivity $a_z = 1.70 \times 10^{-7}$ m^2 s^{-1}. All these measurements lead to the estimation of the thermal conductivities: $\lambda_x = \lambda_z = 0.103$ W m^{-1} K^{-1} and $\lambda_y = 0.190$ W m^{-1} K^{-1} with an estimated precision of 5%.
- An anisotropic thin super-insulating material: a fiber reinforced aerogel available in sheet with a thickness of 9 mm. The fibers are oriented in the plane Oxy so that it may be expected that $\lambda_x = \lambda_y$ and $\lambda_z < \lambda_x$. To our knowledge, the classical methods do not apply for the measurement of the conductivities λ_x and λ_y of this type of samples.

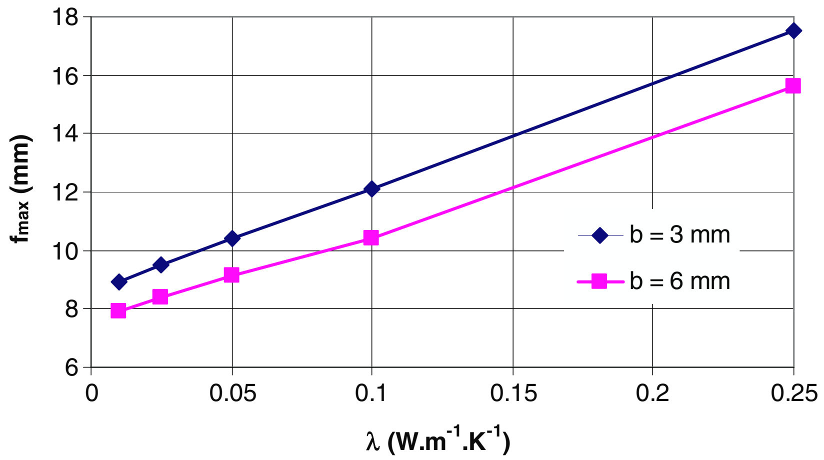

Figure 9. Sample thickness f_{max} corresponding to an estimation error of 1% on λ for a square section sample of 40 mm width.

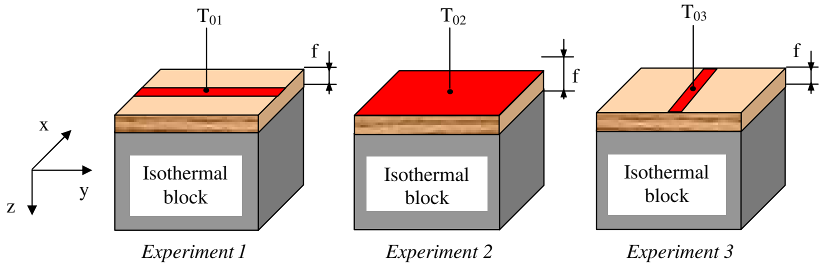

Figure 10. Schema of the three experiments carried out.

Table 2. Results of the hot strip measurements realized on polyethylene foam.

	Measurement number	ϕ_0 (W m^{-2})	ΔT (K)	λ (W m^{-1} K^{-1})
$f = 5.2$ mm	1	76.3	6.17	0.0443
	2	109.9	8.89	0.0443
	3	149.6	12.05	0.0444
	Mean value			0.0443

This study has been realized using two different hot strips.

- A resistive heating element with a thickness of 0.25 mm, length of 44.0 mm and width of 6.5 mm.
- A resistive heating element with a thickness of 0.25 mm, length of 44.0 mm and width of 44.0 mm.

A type K thermocouple made of two wires having a diameter of 0.03 mm has been fixed at the center of the hot strip.

Three experiments have been realized on the polyethylene foam sample with dimensions 44.0 × 44.0 × 5.2 mm^3 using the hot strip having a 6.5 mm width. The results are presented in table 2.

The mean thermal conductivity estimated using the simplified model is $\lambda = 0.0443$ W m^{-1} K^{-1} deviating from the estimated value with the three-layer device ($\lambda = 0.0425$ W m^{-1} K^{-1}) only by 4.2%.

It has been verified using figure 9 that the hypotheses of the simplified model are verified for this polyethylene foam sample with a 5.2 mm thickness and a thermal conductivity greater than 0.04 W m^{-1} K^{-1}.

For Ayous samples, three experiments have been carried out for each configuration represented in figure 10. The dimensions of the samples were surface area $S = 44 \times 44$ mm^2 and thickness $f = 8.0$ mm. Experiments 2 and 3 have been realized with the hot strip of 6.5 mm width. Experiment 2 has been realized with the hot strip having an area of 44 × 44 mm^2 identical to the area of the samples corresponding to the center hot plate method (Jannot *et al* 2010a). It can be noted in figure 10 that the directions Ox and Oz are perpendicular to the fiber direction since the Oy direction is parallel to fibers.

Table 3 presents the experimental results of the three measurements realized for each configuration. Table 4 presents the values of the thermal conductivities λ_z and λ_x calculated from the results T_{01} and T_{02} of experiments 1 and 2 using relations (33) and (34) of the simplified 2D model. The value λ_y is calculated from the result T_{03} of experiment 3 using relation (35) of the simplified 2D model in which λ_z is considered to be known. The values of the thermal conductivities obtained by the combination of the results of the hot plate and of the flash methods measurements are also presented in table 4. It is noticeable that the difference between the thermal conductivities is less than 10% for λ_x and λ_y, and less than 1% for λ_z. Considering relative uncertainties estimated around 1% for ϕ, f, T_{01}, T_{02} and b_1, the use of relations (40) and (41) leads to a maximum relative error of 14.3% for λ_x and λ_y and of 2.5% for λ_z. These estimated uncertainties are slightly superior to the observed values. Using a sample having a width of 40 mm and a thickness equal to the maximum value $f = 12$ mm given in figure 9 would lead to a minimum uncertainty of 13.6% for λ_x.

Table 3. Experimental results.

	Measurement 1		Measurement 2		Measurement 3	
Sample dimensions			$8 \times 44 \times 44$ mm^3			
Hot strip dimensions	6.5×44 mm^2		44×44 mm^2		6.5×44 mm^2	
Experiment number	ϕ	T_{01}	ϕ	T_{02}	ϕ	T_{03}
1	188.9	7.63	97.03	7.65	251.2	9.11
2	242.7	9.83	144.9	11.33	294.8	10.77
3	303.2	12.34	202.4	15.6	341.9	12.52

Table 4. Values of λ (W m^{-1} K^{-1}) estimated with the hot strip and with other methods.

Hot strip			Hot plate + flash		
λ_z	λ_x	λ_y	λ_z	λ_x	λ_y
0.104	0.094	0.184	0.103	0.103	0.190

It has been verified using figure 9 that the hypotheses of the simplified model are verified for this Ayous sample with 6 mm thickness and thermal conductivity greater than 0.1 W m^{-1} K^{-1}.

The measurements realized for Ayous have been repeated for the super-insulating material using the same hot strips and realizing the same experiments. The following results were obtained: $\lambda_z = 0.015$ W m^{-1} K^{-1} and $\lambda_x = \lambda_y = 0.019$ W m^{-1} K^{-1}, showing the anisotropy of the material.

It is also noticeable that for this thin fibrous material, the hot plate method and the other classical methods could not be used for the estimation of the thermal conductivities λ_x and λ_y.

5. Conclusion

The classical thermal characterization methods are not suited to the measurement of the in-plane thermal conductivity of thin insulating materials. This study presents a simple stationary method based on a hot strip device enabling the estimation of the three directional thermal conductivities of an anisotropic thin insulating material. The simplicity of the method is based on the use of a polynomial expression of a geometrical factor. The validity domain of the 2D model considered for establishing this simplified method was studied. The application of the method to a polyethylene foam lead to a thermal conductivity estimation close (deviation less than 5%) to the value measured with another method.

A derived method designed to estimate the directional conductivities of a thin anisotropic material from three hot strip stationary experiments was also described. This method was applied to a wood and the estimated thermal conductivities were in good agreement with the values obtained by other measurement methods. The method was then applied on a thin super-insulating fibrous material whose in-plane thermal conductivities cannot be measured with the classical methods. The results confirmed the anisotropy of this material. Therefore it was shown that the uncertainty on the two transverse conductivities λ_x and λ_y is at least three times greater than the uncertainty on the perpendicular conductivity λ_z.

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