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Measurement and modelisation of the thermal conductivity of a wet composite porous medium: Laterite based bricks with millet waste additive

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HIGHLIGHTS

- Earth based bricks with millet additive are used for building in Senegal.
- Thermal conductivities are measured for different water and millet contents.
- A new serial–parallel model for effective thermal conductivity of composite wet porous materials is proposed.
- Experimental results are shown to be in good agreement with the model.

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ABSTRACT

Millet waste is traditionally and empirically mixed with laterite for bricks fabrication in Sahelian countries, particularly in Senegal. The aim of this paper was to characterize the thermal conductivity of these bricks as a function of their water and millet contents. Samples having five different millet mass contents Y (from 0 to 0.122 $kg_{mi}\,kg_{la}^{-1}$) with dimensions $10 \times 10 \times 3\ cm^3$ were first fabricated. An original asymmetrical hot plate device was developed and modeled to measure the thermal conductivity of these samples, with their water content varying from 0 to a maximum value of 0.1 $kg_w\,kg_{dm}^{-1}$. An adapted device was developed to prevent water evaporation on the lateral faces of the samples. A new model based on a physical approach of the repartition of air and water inside the solid structure was built. It leads to a more accurate representation than other classical models of the measured variations of the thermal conductivity with the water and millet contents.

1. Introduction

Mixing an insulating material with a building material is one of the simplest process to improve its thermal insulating properties. This process is traditionally used in several Sahelian countries particularly in Senegal where millet waste is added to laterite to make bricks used as building materials. Millet waste that can be seen in Fig. 1 is a very low apparent density material so that its thermal conductivity must be quite low. It is available in great quantities at a very low cost.

Some studies concerning the thermal conductivity of earth-based materials have already been published. Bouguerra et al. [1] studied the influence of the wood content on the thermal properties of wood cement-clay based composites. Nevertheless, the influence of the water content was not investigated. Adam and Jones [2] studied the thermal conductivity of stabilized soil building blocks but they did not investigate the influence of the water content. Meukam et al. [3] studied the evolution of the thermal conductivity of stabilized soil building blocks with pouzzolane or sawdust addition as a function of the water content. Nevertheless, no interpretation of the results based on the structure of the material was presented and no predicting model was proposed. Khedari et al. [4] studied the thermal conductivity of coconut fiber-based soil–cement blocks and Omubo-Pepple et al. [5] studied cement stabilized lateritic bricks with sea shell addition but the influence of the water content was not investigated in these two studies. The same remark may be done concerning the work of Goodhew and Griffiths [6] concerning unfired clay bricks with straw and wood chippings.

The variations of the thermal conductivity of a porous medium with its water content have already been studied by several authors. Azizi et al. [7] and Tong et al. [8] developed a model based on the Krischer model [9] for a wet porous medium. Ochs et al. [10] proposed a refinement of the Krischer's model by taking into

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Nomenclature

a	thermal diffusivity ($m^2\,s^{-1}$)
c	specific heat ($J\,kg^{-1}\,K^{-1}$)
C_h	thermal capacity of the heating element per area unit ($J\,m^{-2}\,K^{-1}$)
e	thickness (m)
E	thermal effusivity ($W\,m^{-2}\,K^{-1}\,s^{1/2}$)
m	mass (kg)
p	Laplace parameter
Rc	thermal contact resistance between the heating element and the sample ($m^2\,K\,W^{-1}$)
t	time (s)
T	temperature (K)
V	volume (m^3)
X	dry basis water content ($kg_w\,kg_{dm}{}^{-1}$)
Y	millet mass content ($kg_{mi}\,kg_{la}{}^{-1}$)
λ	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
ε	porosity
ρ	density ($kg\,m^{-3}$)
ϕ	heat flux density ($W\,m^{-2}$)
Φ	Laplace transform of the heat flux density
θ	Laplace transform of the temperature

Subscripts

a	air
dm	dry matter
exp	experimental
g	grain
h	heating element
i	interstice between two grains
la	laterite
mi	millet waste
mod	model
s	solid phase
v	void volume around the grain
w	water
0	insulating blocks

account the opened pores and the closed pores inaccessible for water. Woodside and Messmer [11] proposed a weighted geometric mean of the parallel and series model.

Other authors presented analytical models leading to quite complex expressions of the thermal conductivity such as Sarwar and Majumdar [12].

In a theoretical study, Wang et al. [13] proposed to represent complex materials as composites made of materials perfectly represented by one of the five following models: series, parallel, two forms of Maxwell–Eucken and Effective medium. No experimental validation is presented.

The first aim of this study was first to estimate how much the mixing of millet waste with laterite modifies the thermal conductivity compare to pure laterite bricks as done by several authors for earth based materials. The variation of their thermal conductivity with the water content X and the millet content Y will also be determined experimentally as done by Meukam et al. [3]. The formerly quoted models will be tested to represent the experimental data leading to unsatisfying agreement between theoretical and experimental values. A new model based on a physical approach of the repartition of air and water inside the solid will finally be developed and tested leading to satisfying results.

2. Experimental devices and principle of the methods

2.1. Samples preparation

The laterite powder used was extracted directly from the soil in the region of -Matam in north Senegal. The raw laterite was sieved so that the maximum grain size was 1 mm and then it was kept into sealed recipients. The laterite powder is first mixed with a chosen quantity of millet waste. Then water is added until mixing lead to a homogeneous paste. This paste is pressed in a mould with internal dimensions $10 \times 10 \times 3\ cm^3$ with a constant pressure around 1 bar. After removal from mould, samples are set into seal plastic bags for several days to obtain an uniform water content. A first thermal conductivity measurement is realized then the sample is removed from the bag and exposed to room air temperature and humidity while its decreasing mass is controlled. When its mass has reached a chosen value, the sample is set again into seal plastic bags for several days to obtain an uniform water content. The process is repeated at least four times before the samples are placed for 3 days in a vacuum chamber in which the pressure is lowered below 10^{-2} mbar. After being weight to measure their dried mass, a last thermal conductivity measurement is done with each dried samples, their porosities are also measured with a pycnometer described elsewhere by Bal et al. [14].

2.2. Thermal conductivity measurement method

The thermal conductivity was measured using a hot plate derived method previously used to measure thermal capacity [14]. Since it is difficult to obtain two identical samples having exactly the same water content, an asymmetrical experimental device represented in Fig. 2 was chosen.

A plane heating element having the same section ($10 \times 10\ cm^2$) as the sample is placed under the sample. A type K thermocouple made with two wires with a 0.005 mm diameter is stuck on the lower face of the heating element. This disposal is placed between two extruded polystyrene blocks with a thickness 5 cm set between two aluminum blocks with a thickness 4 cm. A heat flux step is sent into the heating element and the transient temperature $T(t)$ is recorded. Since the thermocouple is in contact with polystyrene that is a deformable material, the presence of the thermocouple does not increase the thermal contact resistance between the heating element and the polystyrene. Furthermore, since polystyrene is an insulating material, this thermal contact resistance will be neglected.

The system is modeled with the hypothesis that the heat transfer remains unidirectional (1D) at the center of the sample during the experiment. This hypothesis will be verified by a 3D simulation realized with COMSOL and by the analysis of the residues of estimation: difference between the modeled 1D transient temperature $T_{mod}(t)$ and the experimental temperature $T_{exp}(t)$.

Fig. 1. Pictures of millet waste used as insulating material and of a sample.

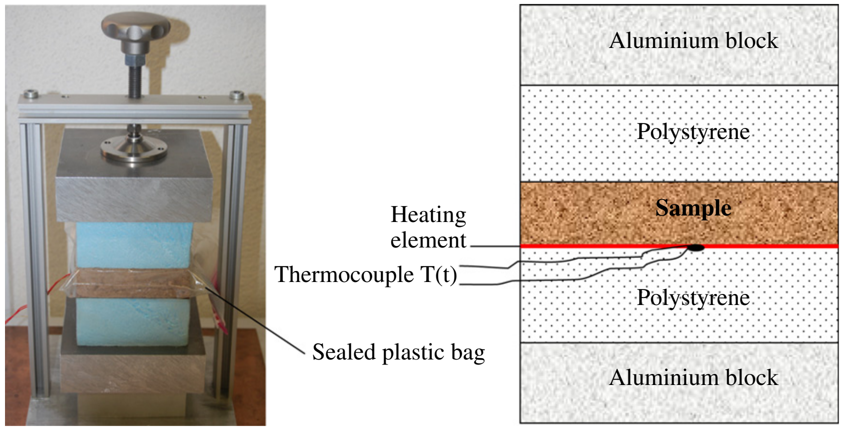

Fig. 2. Schema and view of the experimental hot plate device.

Nevertheless, since wet materials have to be characterized, the problem of surface water evaporation must be addressed. Without special care, the evaporation that will occur on the lateral face of the heated sample will increase the convection heat transfer coefficient. The result would be that the time during which the heat transfer at the center remains 1D would be shortened. To avoid this problem, the samples have been placed in sealed thin plastic bags (polyethylene with a thickness 0.05 mm) in which the air reaches an equilibrium humidity with the sample, preventing surface evaporation. This device can be seen on the view in Fig. 2. It has been verified that the thermal resistance of the plastic bag is negligible compared with the samples thermal resistance.

Within these hypotheses, one can write the following quadrupolar matrix relation [15]:

$$\begin{bmatrix}\theta\\ \Phi_{01}\end{bmatrix}=\begin{bmatrix}1 & 0\\ C_hp & 1\end{bmatrix}\begin{bmatrix}1 & Rc\\ 0 & 1\end{bmatrix}\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}A_0 & B_0\\ C_0 & D_0\end{bmatrix}\begin{bmatrix}0\\ \Phi_1\end{bmatrix}=\begin{bmatrix}A_1 & B_1\\ C_1 & D_1\end{bmatrix}\begin{bmatrix}0\\ \Phi_1\end{bmatrix} \quad (1)$$

$$\begin{bmatrix}\theta\\ \Phi_{02}\end{bmatrix}=\begin{bmatrix}A_0 & B_0\\ C_0 & D_0\end{bmatrix}\begin{bmatrix}0\\ \Phi_2\end{bmatrix} \quad (2)$$

$$\text{with}: \ \Phi_0=\frac{\phi_0}{p}=\Phi_{01}+\Phi_{02} \quad (3)$$

$\theta(p)$ is the Laplace transform of the temperature $T(t)$, p is the Laplace parameter, Φ_{01} is the Laplace transform of the heat flux density living the heating element (upstream), Φ_{02} is the Laplace transform of the heat flux density living the heating element (downstream), Φ_0 is the Laplace transform of the total heat flux density produced in the heating element, φ_0 is the heat flux density produced in the heating element, C_h is the thermal capacity of the heating element per area unit: $C_h = \rho_h c_h e_h$, Rc is the thermal contact resistance between the heating element and the sample, Φ_1 is the Laplace transform of heat flux density input on the upper aluminum block and Φ_2 is the Laplace transform of heat flux density input on the lower aluminum block.

$$A=D=\cosh\left(\sqrt{\frac{p}{a}}e\right);\quad B=\frac{\sinh\left(\sqrt{\frac{p}{a}}e\right)}{\lambda\sqrt{\frac{p}{a}}};\quad C=\lambda\sqrt{\frac{p}{a}}\sinh\left(\sqrt{\frac{p}{a}}e\right) \quad (4)$$

$$A_0=D_0=\cosh\left(\sqrt{\frac{p}{a_0}}e_0\right);\quad B_0=\frac{\sinh\left(\sqrt{\frac{p}{a_0}}e_0\right)}{\lambda_0\sqrt{\frac{p}{a_0}}};\quad C_0=\lambda_0\sqrt{\frac{p}{a_0}}\sinh\left(\sqrt{\frac{p}{a_0}}e_0\right) \quad (5)$$

λ is the sample thermal conductivity, a is the sample thermal diffusivity, e is the sample thickness, λ_0 is the polystyrene thermal conductivity, a_0 is the polystyrene thermal diffusivity and e_0 is the polystyrene thickness.

$$\text{This system leads to}: \ \theta(p)=\frac{\Phi_0(p)}{\frac{D_1}{B_1}+\frac{D_0}{B_0}} \quad (6)$$

The principle of the method is to estimate the value of the thermal effusivity $E=\sqrt{\lambda\rho c}$ and of the thermal capacity ρ_c of the sample that minimize the sum of the quadratic error $\Psi=\sum_{i=1}^{N}[T_{\exp}(t_i)-T_{\text{mod}}(t_i)]^2$ between the experimental curve and the theoretical curve calculated with relation (6). The inverse Laplace transformation is realized by use of the De Hoog algorithm [16].

The estimation has been done on a time interval [0, $t_{\max}$] such as the heat transfer at the center of the sample remains 1D until $t_{\max}$. The abacus presented by Bal et al. [14] for this purpose has been used for the determination of $t_{\max}$.

The value of the thermal capacity C_h of the heating element and of the plastic bag is estimated from three symmetrical center hot plate measurements [17] realized with two samples of polystyrene with a thickness of 5 cm.

The thermal conductivity λ is deduced from the values of the thermal effusivity E and of the thermal capacity ρc by : $\lambda=\frac{E^2}{\rho c}$

3. Thermal conductivity models

A composite material composed of a solid phase (s), of water (w) and of air (a) is considered with the solid phase composed with laterite (la) and millet (m). Its composition is defined by the following parameters:

$$-\ \text{Dry basis water content}: \ X=\frac{m_w}{m_s} \quad (8)$$

$$-\ \text{Global porosity of the dried material } (X=0): \varepsilon=\frac{V_a}{V_s+V_a} \quad (9)$$

The contents of laterite and millet in the solid phases are defined by the millet mass content:

$$Y=\frac{m_{mi}}{m_{la}} \quad (10)$$

According to Wiener [18], the lowest possible value of the thermal conductivity is given by the series model and the highest is given by the parallel one:

– The series model:

$$\lambda=\frac{1}{\frac{\varepsilon_s}{\lambda_s}+\frac{\varepsilon_a}{\lambda_a}+\frac{\varepsilon_w}{\lambda_w}} \quad (11)$$

where $\varepsilon = \varepsilon_a + \varepsilon_w$

– The parallel model:

$$\lambda=\varepsilon_s\lambda_s+\varepsilon_a\lambda_a+\varepsilon_w\lambda_w \quad (12)$$

It has been further shown by Hashin and Shtrikman [19] that for isotropic mixtures, the effective thermal conductivity is independent of pore structure and a refined analysis lead to the Hashin–Shtrikman's bound adapted by Tong et al. [8] to a three phase mixture as:

$$\lambda_{\min}=\lambda_a+\frac{3\lambda_a[\varepsilon_w/(1+f_{w-a})+\varepsilon_s/(1+f_{s-a})]}{\varepsilon_a+\varepsilon_w f_{w-a}/(1+f_{w-a})+\varepsilon_s f_{s-a}/(1+f_{s-a})} \quad (13)$$

$$\lambda_{\max}=\lambda_s+\frac{3\lambda_s[\varepsilon_w/(1+f_{w-s})+\varepsilon_a/(1+f_{a-s})]}{\varepsilon_s+\varepsilon_w f_{w-s}/(1+f_{w-s})+\varepsilon_a f_{a-s}/(1+f_{a-s})} \quad (14)$$

where $f_{w-a} = \frac{3\lambda_a}{\lambda_w-\lambda_a}$; $f_{s-a} = \frac{3\lambda_a}{\lambda_s-\lambda_a}$; $f_{w-s} = \frac{3\lambda_s}{\lambda_w-\lambda S}$; $f_{a-s} = \frac{3\lambda_s}{\lambda_s-\lambda_s}$.

Several authors proposed to estimate the effective thermal conductivity of a mixture by a more or less complicated function of the parallel model and of the series model:

– The Beck's model [20]:

$$\lambda = \sqrt{\lambda_{Series} \times \lambda_{//}} \tag{15}$$

– The Krischer's model [9]:

$$\lambda = \frac{(\lambda_{\text{Series}})(\lambda_{//})}{A(\lambda_{\text{Series}}) + (1-A)(\lambda_{//})} \tag{16}$$

where A is constant depending on the material.

– The Woodside and Messmer's model [11]:

$$\lambda = (\lambda_{\text{Series}})^{\alpha}(\lambda_{//})^{1-\alpha} \tag{17}$$

Ingersoll [21] proposed a more physical model in which water in a parallel arrangement with air is considered in series with the solid structure:

$$\lambda = \left(\frac{1-\alpha}{\lambda_s} + F\frac{\alpha}{\lambda_{a,w}}\right)^{-1} \tag{18}$$

$\lambda_{a,w}$ is the conductivity of air and water corresponding to a parallel arrangement, F and α are adjustable factors.

3.1. Proposed model

The composite material is considered as solid grains in contact with air and liquid water filling the vacuum volume. The grains are considered weakly porous with an internal porosity ε_g filled with air and liquid water. The wide variation of the thermal conductivity with the water content leads us to assume that the contact resistance between two grains may be strongly reduced by the presence of liquid water. We thus proposed the equivalent schema represented in Fig. 3.

The following notations will be used:

– V is the elementary volume.
– V_{gv} is the void volume inside the grain.
– V_{gw} is the volume of water inside the grain.
– V_{ga} is the volume of air inside the grain.
– V_i is the volume of the interstice.
– V_{iw} is the volume of water in the interstice.
– V_{ia} is the volume of air in the interstice.
– V_v is the volume of the void volume.
– V_{vw} is the volume of water in the void volume.
– V_{va} is the volume of air in the void volume.
– V_s is the volume of the solid phase.
– e is the global porosity.
– ε_g is the internal grain porosity.

Another hypothesis of the model is that the volume fraction occupied by the water is the same in the interstice and in the void volume so that:

$$\alpha = \frac{V_i}{V_Y} = \frac{V_{iW}}{V_{YW}} = \frac{V_i}{V_W} \tag{19}$$

where α is the ratio between the volume of the grain interstice V_i and the external void volume V_v. The equivalent thermal conductivity of the grain is:

$$\lambda_g = \frac{\lambda_s V_s + \lambda_w V_{gw} + \lambda_a V_{ga}}{V_s + V_{gw} + V_{ga}} \tag{20}$$

The equivalent thermal conductivity of the interstice and of the vacuum space is:

$$\lambda_v = \lambda_i = \frac{\lambda_w V_{iw} + \lambda_a V_{ia}}{V_{iw} + V_{ia}} = \frac{\lambda_w V_{vw} + \lambda_a V_{va}}{V_{vw} + V_{va}} \tag{21}$$

The equivalent thermal conductivity of the grain in series with the interstice is:

$$\lambda_{i+g} = \frac{V_i + V_g}{\frac{V_i}{\lambda_i} + \frac{V_g}{\lambda_g}} \tag{22}$$

Finally, the equivalent thermal conductivity of the elementary volume is:

$$\lambda_{\text{mod}} = \frac{V_{i+g}\lambda_{i+g} + V_v\lambda_v}{V_i + V_g + V_v} \tag{23}$$

Considering a cell with a global volume V and with m_{la} = 1 kg, m_{mi} = 1 + Y and m_w = (1 + Y)X, the different volumes are calculated as follows:Global volume:

$$V = \frac{(1+X)(1+Y)}{\rho_s(X,Y)} \tag{24}$$

$$\text{Solid volume}: V_s = (1-\varepsilon)V \tag{25}$$

$$\text{Void volume in the grain}: V_{gv} = \frac{\varepsilon_g V_s}{1-\varepsilon_g} \tag{26}$$

where ε_g in the internal porosity of the grain.

The total volume of water is:

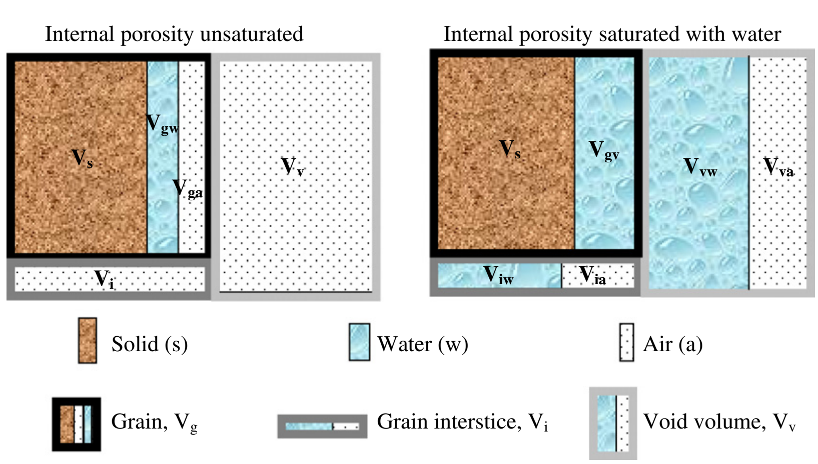

Fig. 3. Model of the elementary volume of the composite wet material.

$$V_w = \frac{X(1+Y)}{\rho_w} \tag{27}$$

The volume of water inside the grain is:

$$V_{gw} = V_w \text{ if } V_{gv} > V_w \tag{28}$$

$$V_{gw} = V_g \text{ if } V_{gv} < V_w \tag{29}$$

The volume of air inside the grain is:

$$V_{ga} = V_{gv} - V_w \tag{30}$$

The volume of water outside the grain is:

$$V_{vw} + V_{iw} = V_w - V_{gw} \tag{31}$$

Using relation (19), the volume of water in the void volume may be calculated as:

$$V_{vw} = \frac{V_w - V_{gw}}{1+\alpha} \tag{32}$$

The volume of air outside the grain is:

$$V_{va} + V_{ia} = \varepsilon V - V_g - (V_w - V_{gw}) \tag{33}$$

Using relation (19), the volume of air in the void volume may be calculated as:

$$V_{va} = \frac{\varepsilon V - V_g - (V_w - V_{gw})}{1+\alpha} \tag{34}$$

All the previously defined volumes can be calculated using relation (19) and relations (24)–(34) if the values of the following parameters are known: ε, ε_g, α, X, Y, ρ_s, ρ_w. Then the modeled thermal conductivity λ_{mod} can be calculated using relations (20)–(23) if the parameter λ_s is known.

The porosity εY and the intrinsic densities ρ_s, ρ_w has been previously measured with a pycnometer [14]. The unknown parameters of the models that must be identified are thus: λ_s, ε_g and α.

4. Results and discussion

The thermal conductivity measurement method was first applied to a PVC sample which properties have been measured by the flash method [22] and the tiny hot plate method [23]: $a = 1.25 \times 10^{-7}\ \mathrm{m^2\,s^{-1}}$ and $\lambda = 0.184\ \mathrm{W\,m^{-1}\,K^{-1}}$ leading to $\rho_c = 1.47 \times 10^6\ \mathrm{J\,m^{-3}\,K^{-1}}$. The PVC sample dimensions were $0.59 \times 10 \times 10\ \mathrm{cm^3}$. The experiment lead to the following results: $\lambda = 0.184\ \mathrm{W\,m^{-1}\,K^{-1}}$ and $\rho_c = 1.39 \times 10^6\ \mathrm{J\,m^{-3}\,K^{-1}}$, so that the deviations with the previously known values are lower than 4.1% that is quite acceptable. This result validates the measurement method with a precision better than 5%.

The apparent density of the five samples varied between $1950\ \mathrm{kg\,m^{-3}}$ for the pure laterite dry block and $1180\ \mathrm{kg\,m^{-3}}$ for the dry block with the maximum millet mass content $Y = 0.122\ \mathrm{kg_{mi}\,kg_{la}}^{-1}$. The corresponding values of the thermal conductivities varied between 1.4 and $0.29\ \mathrm{W\,m^{-1}\,K^{-1}}$. This result already demonstrates the interest of adding millet waste to lower the thermal conductivity of laterite based bricks.

As described in the paragraph "Sample preparation", the thermal conductivity of each of the five samples (with a millet mass content varying from $Y = 0$ to $Y = 0.122\ \mathrm{kg_{mi}\,kg_{la}}^{-1}$) have been measured for at least five different water contents X varying between the maximum value obtain after molding and a null value. As an example, Fig. 4 represents the experimental values of the thermal conductivity of sample having a millet mass content $Y = 0.0611\ \mathrm{kg_{mi}\,kg_{la}}^{-1}$ for six different values of the water content (between 0 and $0.054\ \mathrm{kg_w\,kg_{dm}}^{-1}$).

Two main remarks can be made:

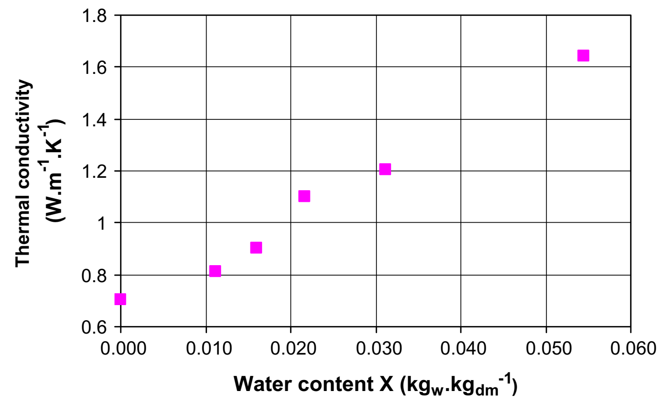

Fig. 4. Experimental result for thermal conductivity of a sample with $Y = 0.0611\ \mathrm{kg_{mi}\,kg_{la}}^{-1}$.

- The variation of the thermal conductivity λ is very important compared to the weak variation of the water content X: λ is multiply by 2.3 when X grows from 0 to $0.054\ \mathrm{kg_w\,kg_{dm}}^{-1}$.
- The thermal conductivity increases slowly with the water content X for the low values of X ($X < 0.1\ \mathrm{kg_w\,kg_{dm}}^{-1}$) and then more rapidly for the higher values.

The first remark leads us to consider that the thermal conductivity is affected in a series way by thermal contact resistances (very thin air layer) that can strongly decrease if a small water content is present in this layer.

The second remark leads us to consider that starting from a dried material, a low increase of water content affects weakly the thermal conductivity because the water is not first placed in the thermal contact resistance but must filled the internal porosity of the solid grain. After the internal porosity of the grain is filled, a part of the water is set between the grains leading to a strong decreasing of the thermal contact resistances. The remaining part is mixed with air in the void volume between the lateral faces of the grains.

These two remarks have led us to propose the model previously described and represented in Fig. 3.

The classical models described by relations (11)–(18) were first tested. The parameters of the models represented by relations (15)–(18) were estimated by applying a minimization algorithm to the sum of the quadratic errors between the experimental and modeled thermal conductivity. The experimental results were firstly processed by this way. As an example, Fig. 5 represents the thermal conductivity calculated with each model both with the experimental results for $Y = 0.0611\ \mathrm{kg_{mi}\,kg_{la}}^{-1}$. Table 1 gives the values of the estimated parameters of each model both with the mean deviation between the experimental and the modeled values of the thermal conductivity.

The results show that only the Woodside and Messmer's model and the Krischer's model lead to a satisfying representation of the variation of the thermal conductivity as a function of the water content. Nevertheless, the estimated values of the thermal conductivity of the solid fraction λ_s have not any physical meaning since the obtained values are greater than $100\ \mathrm{W\,m^{-1}\,K^{-1}}$.

Then, the same work has been done with our model. For each of the sample having a millet mass content Y, a minimization algorithm has been used to estimate the following parameters:

- The thermal conductivity λ_s of the solid phase (laterite + millet).
- The ration α between the volume of the interstice and the volume of the external void.
- The internal porosity ε_g of the grain.

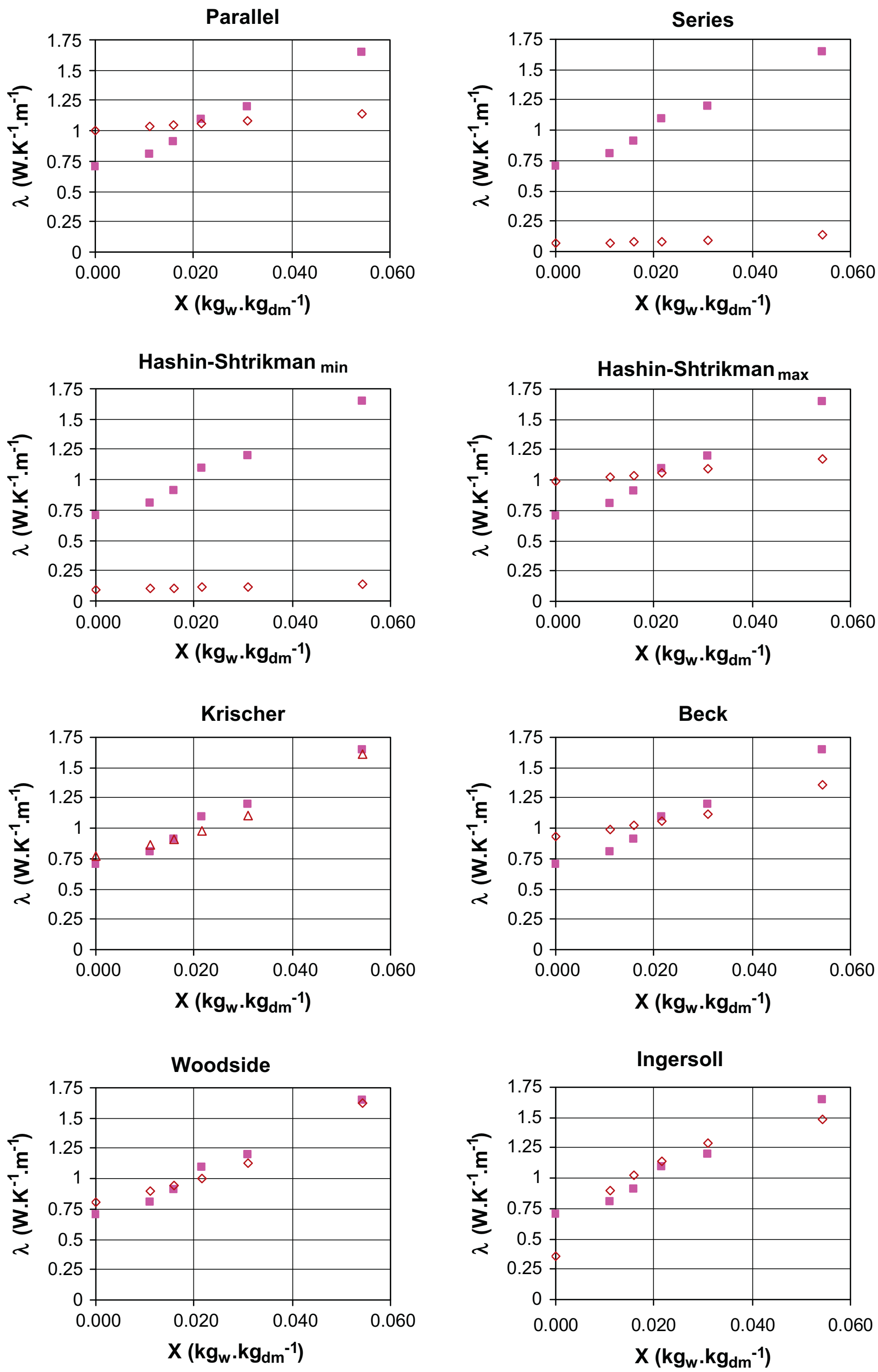

Fig. 5. Experimental thermal conductivity (■) and values calculated with different models (◇) for $Y = 0.0611$ kg_{mi} kg_{la}^{-1}.

Table 1
Estimated parameters of the models and mean relative deviation between experimental and modeled thermal conductivities for $Y = 6.11$ kg_{mi} kg_{la}^{-1}.

	Parallel	Series	HS_{min}	HS_{max}	Beck	Woodside	Krischer	Ingersoll
λ_s (W m^{-1} K^{-1})	1.63	8.6 10^7	0.79	1.90	21.7	3.0 10^{19}	141	1.88
α	–	–	–	–	–	0.947	0.0868	0.034
F	–	–	–	–	–	–	–	1.72
Deviation (%)	21.7	91.4	88.7	20.5	16.2	7.8	6.7	15.5

The values of the porosity of each sample has been measured previously (14) both with the intrinsic densities of the laterite and of the millet using a pycnometer. The values of the porosities are reported in Table 1, the measured values of the intrinsic densities are: $\rho_{la} = 2759$ kg m^{-3} and $\rho_{mi} = 1164$ kg m^{-3}. The results are presented in Table 2.

Table 2
Estimated values of the different parameters.

Y ($kg_{mi}\ kg_{la}^{-1}$)	Porosity ε	λ_s ($W\ m^{-1}\ K^{-1}$)	α	ε_g	Mean relative deviation (%)
0	0.29	4.86	0.0244	0	6.2
0.0305	0.325	4.63	0.0442	0.0246	7.6
0.061	0.39	3.57	0.0297	0.0224	4.0
0.0916	0.428	3.27	0.0390	0.0276	4.8
0.122	0.524	3.00	0.0304	0.0244	4.7

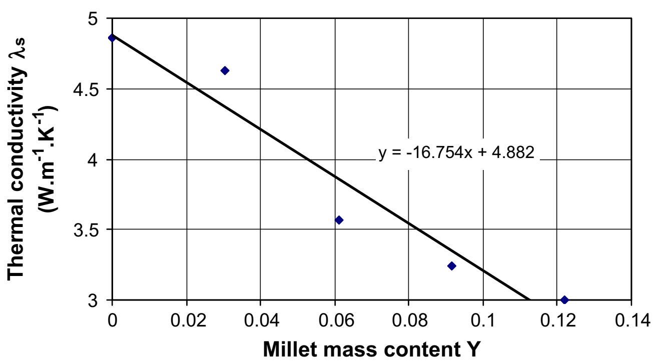

Fig. 6. Solid thermal conductivity as a function of the millet mass content Y.

Table 3
Mean relative deviation (%) between the experimental and the theoretical values of the thermal conductivity.

Y ($kg_{mi}\ kg_{la}^{-1}$)	$\lambda_{scalculated}$ ($W\ m^{-1}\ K^{-1}$)	α	ε_g	Mean relative deviation (%)
0	4.88	0.0244	0	6.3
0.0305	4.37	0.0358	0.0248	8.3
0.061	3.86			8.9
0.0916	3.35			5.6
0.122	2.83			9.4

As expected, it can be noticed that the thermal conductivity of the solid decreases when the millet water content increases. It may be represented approximately by a straight line as shown in Fig. 6 and the thermal conductivity of a dry sample of earth based laterite brick with a mass fraction Y of millet may be approximated by the relation:

$$\lambda_s = 4.88 - 16.75Y \tag{35}$$

It is also remarkable that the estimated values of the ratio α between the volume of the interstice and the volume of the external void have the same order of magnitude with a mean value of 0.033.

The same remark can be made for the internal porosity ε_g of the solid grain with a mean value of 0.0233 for the samples with millet with an exception for pure laterite sample for which $\varepsilon_g = 0$. This seems to show that the porosity inside the laterit grain is negligible.

The conclusion is that the estimation leads to represent the material by a stack of solid grains having an internal porosity $\varepsilon_g = 0$ of the global porosity for pure laterite and $\varepsilon_g = 0.025$ for samples containing millet. Between the grains, an interstitial layer representing around 0.036 of the external void volume is placed in series with the grain. The remaining void volume is placed in parallel. The water first fills the internal porosity (if non-null) of the grain and then is reported in the ratio 0.036–0.964 between the interstice and the remaining void volume.

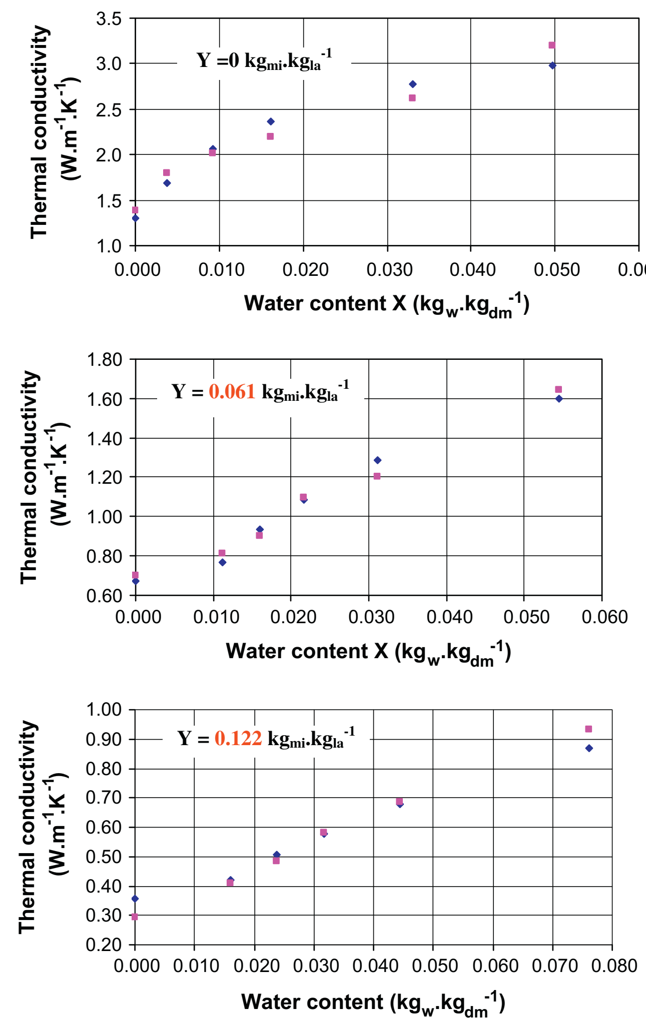

Fig. 7. Experimental and theoretical thermal conductivity as a function of the water content X for several millet mass content Y.

The values of Table 1 has then been used to calculate the modeled thermal conductivity using the relations (19)–(34). Fig. 7 represents the theoretical and the experimental values of the thermal conductivity obtained for the samples with $Y = 0$, 0.0611 $kg_{mi}\ kg_{la}^{-1}$ and 0.122 $kg_{mi}\ kg_{la}^{-1}$. The values are found to be in good agreement with a mean relative deviation of 5.5% between the experimental and the theoretical values for the five samples. The mean relative deviations for each samples are reported in Table 2.

Other theoretical values have been calculated for all the samples using the values of Table 3, i.e. $\alpha = 0.0244$ and $\varepsilon_g = 0$ for $Y = 0$ and the same values $\alpha = 0.0358$ and $\varepsilon_g = 0.0248$ for all the samples with $Y \neq 0$. In all cases, the value of λ_s is calculated by relation (35). The mean relative deviation between the theoretical and the experimental values for each sample with a millet mass fraction Y are reported in Table 3. It may be noticed that the maximum value of the mean relative deviation is 9.4% that is quite acceptable for samples handmade with natural materials.

5. Conclusion

This study has underlined the strong influence of the water content on the thermal conductivity of laterite based bricks. It has also been shown that adding millet waste may strongly decrease their thermal conductivity (from 1.4 for dry pure laterite blocks to 0.29 $W\ m^{-1}\ K^{-1}$ for dry laterite blocks with 0.122 $kg_{mi}\ kg_{la}{}^{-1}$ mass content of millet waste). An adapted model has been conceived to predict the thermal conductivity of the bricks as a function of both water content and millet content. It was found that this model enables to calculate a sufficiently precise value of the thermal conductivity of laterite based bricks as a function of its millet and water contents. The suitability of this model for other buildings material will be further studied.

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